

1982

# Asset market approach to exchange rate determination

Isaac Quao Mensah  
*Iowa State University*

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**ASSET MARKET APPROACH TO EXCHANGE RATE DETERMINATION**

*Iowa State University*

PH.D. 1982

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Asset market approach to  
exchange rate determination

by

Isaac Quao Mensah

A Dissertation Submitted to the  
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## CHAPTER 1. INTRODUCTION

Recently, increased attention has been focused on the asset market approach to exchange rate determination. The asset market approach to exchange rates views an exchange rate as the relative price of national monies. And it is viewed as one of the prices that equilibrates the international markets for various financial assets. Hence, the supplies of and demand for stocks of various monies and other financial assets are the important elements under this approach.

In contrast, the traditional theory of exchange rate determination is based solely on the current account. It focuses on the demand for and the supply of foreign exchange and the price elasticities of import demands and export supplies. Demand for foreign exchange is determined by value of imports, while the supply of foreign exchange is determined by the value of exports; both of these are flow concepts.

Consequently, these two theories view equilibrium exchange rate determination differently. First, the traditional theory views the exchange rate as the relative price of national outputs, instead of as the relative price of national monies. Second, it assumes the exchange rate to be determined by conditions for equilibrium in the

markets for flow of funds, instead of by the conditions for equilibrium in the markets for stocks of assets. In view of the asset market approach, considerations of elasticities is irrelevant, since the traditional theory on which it is based has some erroneous concepts. These are discussed in Chapter 2.

The development of a model capable of explaining exchange rate determination requires the identification of factors that affect exchange rate levels. Previous studies have identified several factors that influence exchange rate levels. First, the conditions of assets market equilibrium play a vital role in determining the exchange rate. The assets aspects of the model arise through the assumption that the exchange rate as the relative price of two assets, is primarily determined by the relative supplies and demand for these assets. Second, the current value of the exchange rate is strongly influenced by expectations of its future value and is dependent on the information that underlies these expectations. Exchange rate expectations are influenced by every conceivable economic, political, social and psychological factor. The exchange rate expectation could be treated as predetermined in the short run, and the rate of exchange determined as a valuation of domestic money relative to foreign assets so as to maintain money market equilibrium.

This approach is used by Frenkel (1976) and Kouri (1976). Alternatively, the exchange rate expectation could be viewed as providing the critical equilibrating mechanism. Under this rule, the expectational variable is made endogenous in the short run, and given interest rate parity is assumed to adjust in such a way that the expected rate of return equilibrates the money market. This is adopted by Dornbusch (1976), Turnovsky and Kingston (1977), and Mussa (1979). Third, the equilibrium exchange rate depends on the current and real factors that affect absolute and relative prices; it also depends on current expectations concerning the future behavior of these exogenous monetary and real factors. Many other factors impinge on the level and adjustments of the exchange rate; for example, differential movements in absolute price levels as suggested by one purchasing power parity doctrine; and changes in the relative price of different national outputs essential to the maintenance of external trade balance equilibrium.

#### Objectives of the Study

In his important contribution, Dornbusch (1976) presents a monetary approach model to exchange rate determination in the short run. In his model, domestic money is the only available asset, there are no other alternative assets, and no wealth effects. An addition

of alternative assets would enlarge the range of analysis to consideration of capital flows, and the determination of domestic interest rates. With the introduction of the effects of the level of domestic holdings of wealth on assets demands, there could be a positive wealth effect on desired holdings of assets. The first order effect of this extension would be an inverse dependence of the exchange rate (defined as the price of a unit of the foreign currency in terms of domestic money), and the price level on the level of wealth. His simple dynamic macroeconomic model is employed to study how exchange rates respond to unanticipated shocks to the economy. This analysis could be extended to include anticipated shocks to the economy, using the framework developed by Fischer (1979).

The purpose of this study then, is to modify and extend Dornbusch's (1976) study. Dornbusch assumed that the expected rate of depreciation of the spot rate is proportional to the discrepancy between the long-run rate and the current spot rate. This expectation formation is purely ad hoc, and therefore this approach will be modified and rational expectations introduced into the model. The introduction of rational expectations merges the assets and current account theories of exchange rates because it can lead to a fully anticipated equilibrium



path in which asset prices adjust in part to reflect future current account developments. Secondly, in this present model, the demand for real money balances is assumed to depend not only on the domestic interest rate and real income, but also on wealth. Finally, in this study, Dornbusch's analysis will be extended to consider the impact of anticipated monetary expansions on the exchange rate.

A model of the financial sector and the goods sector is presented. The model will share the central features of the analytical work of Dornbusch (1976), Dornbusch and Fischer (1980), Mussa (1976 and 1979), Kouri (1976), and others. Specifically the objectives of the study are: (1) to analyze the role of the asset market equilibrium, good market equilibrium and expectations in the determination of the exchange rate in the short run. (2) to analyze the effects of monetary disturbances, particularly to examine the effects of anticipated and unanticipated monetary expansions on the exchange rate. (3) to develop tests of the asset market approach to the determination of exchange rates, using data from the flexible exchange rate period.

The monetary approach model provides several specific conclusions that can be tested empirically. In this model, both monetary and fiscal policies have real effects.

Because money is viewed as an asset, changing the money supply changes real wealth and hence, real expenditures. These have significant effects on the exchange rate level. In the empirical section of this study, an attempt will be made to provide an empirical analysis of some aspects of the asset market approach to exchange rate theory.

### Outline of the Study

The outline of this study is as follows: In Chapter 2, the theoretical treatment of alternative models for explaining short-run movement of exchange rates is discussed. In particular, we take a look at the traditional theory of exchange rate determination, which is based on relative price levels and trade flows; and also we examine the modern theory, which is based on financial-equilibrium models. Much of the survey focuses on the recent developments of the financial-equilibrium models. Some critical evaluation of both theories is also made, to show the relative pros and cons of the theories. The latter part of Chapter 2 focuses on the recent empirical literature and some critiques of the models used.

The theoretical model is presented in Chapter 3. In the first section the general features of a small country model are presented, and the equilibrium exchange rate derived. The equilibrium exchange rate depends on the

expected rate of depreciation of the domestic currency; hence we examine the role of expectations on the movement of the exchange rate, by imposing the requirement of rational expectations. Imposing this requirement, we realize that the current spot rate depends on current variables of the financial and real sectors, as well as on the current expectations concerning the future behavior of these exogenous variables. With this model developed, it is possible to introduce a distinction between anticipated and unanticipated changes in exogenous variables. First, we examine the effects of unanticipated transitory monetary disturbance on the exchange rate. Second, we look at the effects of unanticipated permanent monetary disturbance; third, the effects of anticipated transitory monetary disturbance and finally, the effects of anticipated permanent monetary disturbance on the exchange rate. The principal results are that unanticipated monetary expansion leads to exchange depreciation; and the anticipated monetary expansion causes the exchange rate to continue to depreciate in an exponential manner until the time the actual change occurs. In the second section of Chapter 3, we extend the model from one small country case to the more general two country case, to examine the determinants of a bilateral exchange

rate and also to examine how the small country result of monetary policy are changed, due to interaction between countries.

In Chapter 4, we examine the empirical validity of a simple asset market model of a bilateral exchange rate, using the United States-Germany data, and also United States-Netherlands data.

## CHAPTER 2. LITERATURE REVIEW

## Theories of the Exchange

## Rate Determination

Significant progress has been made in the theoretical analysis of exchange rate determination, since the exchange rates began to float in 1972. Their fluctuations have resembled those of asset market prices, and these have been dominated by factors prevailing in the financial asset markets. Accordingly, attention has been directed toward the role of the conditions in financial assets markets.<sup>1</sup> From this perspective, the exchange rate is viewed as the relative price of different national monies, and is determined by supply and demand conditions of stocks of different national monies and other financial assets.

Early attempts to investigate the determinants of the exchange rate focused on the demand for supply of foreign exchange and the price elasticities of import demands and export supplies. The demand for foreign exchange in this approach is determined by the value of imports and is measured as a flow of foreign money. The supply of foreign

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<sup>1</sup>This was established in the summary remarks to the Stockholm conference on Flexible Exchange Rates and Stabilization Policy contained in *Scandinavian Journal of Economics*, 78, No. 2 (1976): 386-412.

exchange is determined by the value of one's exports and is also measured as a flow of foreign money. The exchange rate is therefore determined by the equilibrium condition that demand for foreign exchange equals supply of foreign exchange.

In the traditional approach, the focus is on the behavior of imports and exports and the capital flows between countries. This approach views the exchange rate as the relative price of national output, as opposed to relative price of national monies in the asset market approach. It also assumes that the exchange rate is determined by the conditions for equilibrium in the market for flows of funds as opposed to conditions for equilibrium in the market for stocks of assets as in the asset market approach. The approach emphasizes covered interest arbitrage, along with commercial hedging and speculation in determining the equilibrium exchange rate. The theory is based on a detailed description of the determinants of the demand for and supply of forward and spot exchange necessitated by each of these three operations in the foreign exchange market.

There are many criticisms of the traditional approach. For example, if we claim that a change in the exchange rate affects the balance of payments because exchange rate changes induce changes in the relative prices of domestic

and foreign goods, then that implies that the exchange rate is the relative price of national outputs. However, if we assume that the domestic and the foreign countries produce identical and tradeable goods, and that purchasing power parity holds for all commodities, then a depreciation of the domestic currency will increase the domestic money price of every good relative to the foreign money price of that good, by the amount of the depreciation. But there is no reason to believe that this change in nominal prices should be associated with any particular change in relative commodity prices. However, there are cases when exchange rate changes have significant effect on relative commodity prices, or that relative price changes affect the balance of payments. But it is rather important to note that these effects must occur through the impact of these nominal price movements on the transactions demand for money. For example, if a devaluation raises the relative price of imports  $q = (SP^*/P)$ , and we assume that a rise in the relative price of imports will reduce imports and raise exports then this will induce expansion of domestic output and a reduction in foreign output. Hence, the transaction demand for domestic money rises and the demand for foreign money falls. Consequently, there will be a flow of exchange reserves from foreign

to domestic country and this is the mechanism by which the balance of payments is affected.

Another source of criticism is that the traditional approach emphasizes the conditions of markets for flow of funds, and the effects of asset flows on asset stocks are neglected. The implication of this is that, under a fixed exchange rate regime a disequilibrium, caused by a disturbance in the relative commodity prices, will lead to a persistent divergence between the flow demand for and the flow supply of foreign exchange. Since the effects of assets flows on asset stocks are neglected, these flows will persist, until one country runs out of exchange reserves. On the other hand, in the asset market approach, the equilibrium condition is that the demand for the stock of each national asset must equal the stock of that asset available. And hence, any observed flows of funds do occur to correct the existing market disequilibria, but are not considered as the basic determinant of equilibrium.

Recently, asset market models have, therefore, been developed to replace the traditional approach models. These asset market models differ in many respects, but the main emphasis is on the requirement that available stocks of national monies and other financial assets must equal stock demands for these assets as a necessary condition



for equilibrium. These models are usually single country models, treating macroeconomic variables in the rest of the world as predetermined.

A variety of assumptions have been made concerning the number and the nature of financial assets and goods. In one group of models, assets denominated in terms of domestic and foreign currency are assumed to be perfect substitutes. Therefore, in the analysis of the determination of equilibrium in the financial markets it is necessary to specify only the demands for and supplies of monies. This is the approach used by Bilson (1978), Dornbusch (1976), Frenkel (1976), Hodrick (1978) and others. In this approach, there is only one instrument of monetary policy, the supply of money. An alternative group of models differs from the monetarist model of exchange rate determination, in that it does not assume that all other assets but monies are perfect substitutes. It is therefore necessary to specify the demands for and supplies of all assets in the portfolio. These normally include a domestic non-traded bond, which is issued by the domestic government and a foreign traded bond which is denominated in foreign currency and pays an exogenously given world rate of interest. This approach is adopted by Kouri (1976), Turnovsky (1976), Turnovsky and Kingston (1977) and

others. In this approach, two forms of monetary expansion, an open market purchase of domestic bonds, and an open market purchase of foreign exchange are considered and compared. Some interesting qualitative results emerge from these models. In the case of an open economy whose residents hold domestic money, bonds denominated in domestic currency units, and bonds denominated in foreign currency units, it is generally established that the short run effects of an expansionary monetary policy depend, firstly, upon how an increase in money supply is created (an open market operation, or by an exchange market operation), and secondly upon the mode of deficit financing employed by the government. An open market purchase of domestic bonds by the monetary authorities causes the domestic interest rate to fall, and also causes a depreciation of domestic currency. The domestic currency will depreciate more: (a) the greater the substitutability between domestic and foreign assets, (b) if the expectation of asset holders is such that there would not be any subsequent appreciation, (c) the smaller is the fraction of domestic wealth held in the form of foreign currency assets. Similarly the effects of exchange market operation depends on asset substitutability, expectational forces and fraction of domestic wealth held in one form of foreign currency assets. However, if the domestic and

foreign assets are perfect substitutes, then sterilized intervention will have almost no impact on interest rates or exchange rates. But a non-sterilized intervention will have the same effects as an open-market operation by domestic monetary authorities. For the monetarist model, a monetary expansion is shown to induce an immediate depreciation in the exchange rate.

There are some major shortcomings of these models; too little attention has been paid to the role of wealth variables, and very little attention paid to the dynamics of such models, and to analyzing the question of the various impacts of different modes of government financing. Incorporating these into the existing models is of interest because these further analyses will provide some significant insights into the time path of the exchange rate, following a change in monetary policy.

Most of the existing literature focuses on the comparative static properties of the model, either in the short run, when all financial assets are assumed to be predetermined, or in the long run steady state when all accumulation has ceased, and capital stock variables converge to long-run equilibrium values. Little emphasis has so far been placed on dynamic macro models embodying one asset market view of exchange rate determination. Numerous dynamic processes are involved in the adjustment

from short run equilibrium to long run equilibrium, for example, accumulation of assets over time, slow adjustment of prices to variations in aggregate demand, and changes in expectation over time. It seems worthwhile to attempt to model this phenomenon, since it will definitely shed some light on the exact movement of exchange rates over time due to monetary disturbances. Among the major contributions to dynamic analysis are the works of Dornbusch (1976), Kouri (1976) and Branson (1976). Dornbusch (1976) developed a theory of exchange rate dynamics under perfect capital mobility, a slow adjustment of goods markets relative to asset markets, and consistent expectations. He focused on how a monetary expansion affects the time paths of the exchange rate, the domestic price level, and the domestic interest rate. He shows that along a perfect foresight path, a monetary expansion causes the exchange rate to depreciate, and also shows that an initial overshooting of exchange rates occur due to the differential adjustment speed of markets. Kouri (1976) developed a simple dynamic model of the determination of the exchange rate. He analyzed the role of monetary asset equilibrium and expectations, and the role of the process of asset accumulation in the determination of the time path from momentary to long-run equilibrium. His model supports the conclusion that monetary expansion

leads to currency depreciation in the short run; and that the dynamic behavior of the exchange rate depends critically on the nature of expectations formation.

Branson (1976) modified Kouri's analysis by endogenizing the interest rate on domestic interest bearing assets, which Kouri assumed to be fixed in his analysis.

These studies in a dynamic framework have led to some interesting theoretical insights about exchange rate fluctuations. A familiar conclusion from each of these models is that monetary expansion leads to exchange rate depreciation. However, there are no clear cut conclusions on the effects of fiscal policy on exchange rates; these effects depend upon how the government deficit is financed. In recent studies, some attention has been devoted to the question of the time path of exchange rates, following a change in monetary policy. Important contributions came from Dornbusch (1976), Kouri (1976), and Branson (1976). Dornbusch and Kouri conclude that the impact effect of a monetary expansion on the spot exchange rate will be that in the short run the exchange rate will overshoot the long run equilibrium exchange rate. Further, Dornbusch shows that overshooting may occur even when exchange rate expectations show perfect foresight. It is clear from Dornbusch's analysis that when domestic goods prices are sticky, the spillover

effect of disequilibrium in the market for domestic goods created by an unexpected monetary disturbance required that the actual change in the exchange rate exceed the change in the equilibrium rate. Mussa (1979) argued that such overshooting behavior occurs only in response to an unanticipated monetary disturbance, and not in response to expected monetary changes or to any form of real disturbance.

Also, Mussa (1979) and Dornbusch (1976) have shown that there are divergences of exchange rates and national price levels from purchasing power parity, during the process of adjustment to a new long run equilibrium following a monetary change that disrupts an initial long-run equilibrium. Mussa in his analysis arrives at the conclusion that the exchange rate plays an essential role in adjusting the relative price of national outputs to actual and expected changes in the real factors that determine the equilibrium values of this relative price; and that such relative price changes are necessarily associated with divergences from purchasing power parity. Secondly, he shows that if the price of domestic goods is sticky; then unexpected changes in the equilibrium value of this price induced by purely monetary disturbances will spill over onto the exchange rate and induce temporary divergences from purchasing power parity.

These results however, are based on simplified models that heavily obscure the underlying economic structure. More elaborate models will help us better appreciate the sensitivity of exchange rates to various economic policies.

#### Empirical Literature

Since exchange rates began to float in 1972, their movements seem to have been dominated by monetary conditions. This has stimulated several attempts to apply the asset market model empirically in order to explain the exchange rates of the major currencies. Most of the studies derive exchange-rate equation by manipulating money market equilibrium conditions. Studies by Bilson (1978), Frenkel (1976), Girton and Roper (1977) and Hodrick (1978) follow this approach.

Frenkel's model is based on the assumption in hyperinflation periods. During the German hyperinflation, for example, relative price movement swamped all other influences on German exchange rates. Over periods of time long enough for ratios of national price indexes to change radically, purchasing power parity may have considerable validity, but has been discredited as a short run hypothesis in more general circumstances. Bilson (1978) presented a model of exchange rate determination

combining elements of the efficient market and monetary approaches to asset markets. The efficient market characteristics consist of purchasing power parity, interest rate parity, the Fisher equation from intertemporal arbitrage in assets, and the rational expectations assumptions on prices and exchange rates.

These empirical analyses, in attempting to explain short run exchange rate determinants, have considered the effects of different monetary policy measures, the impact of foreign monetary variables, and the influence of real sector variables. In addition, there have been some successful attempts to incorporate the effects of expectational forces. Generally, empirical results have been found to be consistent with the asset market approach to exchange rate determination.

There are a number of problems in the estimation process. Empirical portfolio balance models require data on global stocks of outside assets not held by official agencies, broken down by currency denomination. These are data that at present are partially confidential and difficult to assemble. Secondly, availability of statistical data for measures of foreign wealth is not adequate, and hence these variables are dropped from exchange rate equations. There is some question whether this will seriously bias the results of the regression



analysis. Thirdly, the models in existence assume that exchange rates are permitted to float freely, while in fact governments still intervene in foreign exchange markets from time to time in order to achieve a managed float. This suggests that there may be some reverse causality running from exchange rates to money, at least in the short run. This means that the exchange rate model may not be a completely accurate description of existing exchange rate regimes. Finally, quarterly data may not be suitable for testing what is essentially a model of long run equilibrium. Quarterly data are short run data. As such they may be dominated by transitory dynamic adjustment phenomena that are absent in long run static equilibrium. Annual data are more appropriate for testing an equation that is based on assumptions of interest rate parity, monetary equilibrium, goods market equilibrium and purchasing power parity for traded goods, real income exogeneity, and unidirectional causality between money and exchange rates. A solution would be to augment the model with additional equations and variables to represent dynamic adjustment processes. This might permit the specification of short run influence affecting the exchange rate.

A number of unresolved problems still exist in attempting to explain the determinants of exchange rates. First, the asset market equilibrium model does not provide

a complete theory of exchange rates determination, because most of the explanatory variables are endogenous, except in the very short run. Second, the asset market approach does not claim that the exchange rate is determined entirely in the asset market. The exchange rate together with other macroeconomic variables are determined in a general equilibrium framework by the interaction of flow and stock conditions. Therefore, the asset market equilibrium relationship that is used in the analysis may be viewed as a reduced form relationship, chosen as a convenient framework. The model may be too simple to capture all the influences on the exchange rate. The theory underlying the analysis may be correct, even though its empirical form is inadequate to fit the facts.

The preceding paragraphs have pointed to some of the shortcomings of the existing asset market models for analyzing the process of exchange rate determination. There is also scope for additional analysis of the static and dynamic portfolio balance models. More consideration needs to be given to the treatment of the impact of the growth of wealth on the exchange rate movements. It is also necessary to assess both anticipated and unanticipated monetary policy measures within the realm of instruments oriented toward the various targets of the government's economic policy.

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The ideal approach for this study will be to deal with all these problems, but that is impossible in the framework of a tractable analytical model. The focus will be directed to analyzing the role of asset market equilibrium, goods market equilibrium, and expectations in the determination of the exchange rate in the short run, and then to analyze the effects of monetary disturbances; particularly to examine the effects of anticipated and unanticipated monetary expansions on the exchange rate. The empirical analysis will employ the model developed, and data from the flexible exchange rate period.

## CHAPTER 3. THE THEORETICAL MODEL

This chapter presents the basic methodology to be employed in an attempt to explain the effects of monetary policies on the exchange rate. The formulation of the present model reflects the recent developments by Dornbusch (1976), Dornbusch and Fischer (1980), Frenkel (1976), Kouri (1976), Mussa (1976, 1979) and others. The primary objective is to examine theoretically and empirically the interrelationships between monetary policies and the exchange rate. In Section 1, the analysis focuses on a small country, meaning that repercussion effects associated with the monetary policies are ignored. In Section 2, we will expand the framework of the analysis to include repercussion effects in a two country model. The purpose is to obtain a model for the empirical analysis to be carried out in Chapter 4.

General Features of the  
Small Country Model

The analysis in this section focuses on a small country in the sense that the country exerts no influence on the foreign prices of goods and financial assets. The domestic country is at full employment, produces a single tradeable good, prices are flexible and output

is fixed. We have the competitive arbitrage assumption about prices that

$$P = SP^* \quad (1)$$

where  $P$  is the domestic money price of the domestic good,  $S$  is the price of a unit of the foreign currency in terms of domestic money, and  $P^*$  is the foreign currency price of the domestic tradeable good, which by the small country assumption is taken to be exogenously determined.

The model of the financial sector is derived from Dornbusch and Fischer (1980), and Dornbusch's (1976) framework. Dornbusch (1976) assumed that the demand for money is independent of wealth variables. We will deviate somewhat from Dornbusch's approach by incorporating a portfolio balance view of the asset markets and a wealth constraint into the model. We shall assume that two financial assets are held in the portfolios of the domestic private sector; domestic money held only by domestic residents and denominated in domestic currency; and foreign currency denominated bonds which can be held by domestic residents. For simplicity, bonds are taken to be perpetuities. It is also assumed that the monetary authorities of the foreign country

issue the bonds. The foreign bonds are index bonds each yielding one unit of foreign output per unit of time. At each point in time, individuals allocate their stock of real wealth between on the one hand, bonds and on the other hand, money. Following Dornbusch and Fischer (1980), real wealth is then defined as the sum of the real value of money balances and the real bond holdings:

$$W = \frac{M}{P} + \frac{A}{r^*} \quad (2)$$

where  $W$  is the private sector real wealth,  $M$  is the stock of domestic money holdings of the private sector,  $A$  is the number of income streams from assets accruing to the domestic residents, each yielding one unit of foreign output per unit of time, and  $r^*$  is the exogenously determined world real interest rate, and hence  $A/r^*$  is the real bond holdings. The domestic monetary authorities also hold some of these bonds, and conduct open market operation with these bonds.

Let  $r^*$  be the real interest rate on foreign bonds. It is assumed that the expected rate of domestic inflation  $\pi$ , is equal to the expected rate of currency depreciation  $x$ ; that is, prices abroad are assumed fixed

or stable. The assumption of perfect capital mobility together with these relations implies  $r = r^*$ . Then

$$i = r^* + x \quad (3)$$

where  $i$  is the domestic nominal interest rate, and  $x$  is the expected rate of depreciation of the domestic currency. The alternative cost of holding assets denominated in terms of domestic currency is the condition stated in Equation 3.

The demand for money as a proportion of wealth depends negatively on the alternative cost of holding money,  $r^* + x$ , and positively on real output; which reflects transactions demand.

$$\frac{M^d}{P} = \theta(r^* + x, Y)W \quad (4)$$

Substituting Equation 2 into Equation 4 yields

$$\frac{M}{P} = \frac{\theta(r^* + x, Y)}{1 - \theta(r^* + x, Y)} \frac{A}{r^*} \quad (5)$$

and hence

$$\frac{M}{P} = \frac{A}{r^*} \cdot L(Y, r^* + x) \quad L_1 > 0, L_2 < 0 \quad (6)$$



Given the nominal money supply,  $M$ , the price level depends on the value of bond holdings,  $A$ . An increase in the return stream to the perpetuities raises wealth and hence the demand for real balances. Equilibrium in the money market therefore requires a rise in the real money supply given  $x$ . For a given level of  $M$ , the price level  $P$  then has to fall. An increase in real output raises transactions, this requires an increase in the demand for real balances to finance these transactions, and hence leads to a rise in the real money supply to establish money market equilibrium. For a given level of  $M$ , the price level falls in order to achieve equilibrium. A rise in the foreign real rate of return,  $r^*$  reduces the equilibrium real money stock for two reasons, first because it lowers the share of real balances individuals are willing to hold in their portfolio, and secondly, because it lowers the level of real wealth and hence, the desired demand for real balances. For a given stock of money, a reduction in equilibrium real money stock means a rise in the price level.

The dynamics of the system with a flexible exchange rate are given by the condition that the balance of payments is in equilibrium. Hence, excess of income over spending is the rate at which we are acquiring claims on the rest of the world,  $\dot{A}/r^*$ , hence

$$\frac{\dot{A}}{r^*} = Y + A - E\left(Y + A, \frac{M}{P} + \frac{A}{r^*}\right) \quad (7)$$

Where E is real expenditures, and it depends on real income and real wealth.

Specifying the real money demand function in the conventional way, we have

$$\frac{M}{P} = AY^\gamma e^{-\lambda(r^* + x)} (r^*)^{-1} \quad (8)$$

Using Equations 1 and 8, we can solve for the equilibrium exchange rate level specified in logarithms in Equation 9 below; where s is the logarithm of the exchange rate.

$$s = m - a - \gamma y + \lambda r^* + \ln(r^*) + \lambda x - p^* \quad (9)$$

Without any loss of generality, let us assume that

$\lambda r^* + \ln(r^*) = \delta r^*$ , hence Equation 9 becomes

$$s = m - a - \gamma y + \delta r^* + \lambda x - p^* \quad (10)$$

The lower case letters are used to denote logarithm of the variables. The model establishes that an increase in the domestic money supply, holding all other variables constant, causes an excess supply of money which results in an equiproportionate depreciation of the home currency. An increase in the value of foreign bond holdings raises real wealth, and hence the demand for real money balances.

For a given stock of domestic money, the price level falls to maintain money market equilibrium. A fall in the domestic price level leads to an appreciation in the spot exchange rate to maintain purchasing power parity. A rise in real output stimulates the demand for real money balances, and consequently causes a fall in the price level. This leads to exchange rate appreciation to maintain purchasing power parity. An increase in the foreign real interest rate reduces real balances, raises the domestic price level, and hence causes a depreciation of the exchange rate. An increase in the expected rate of depreciation raises the domestic interest rate, and hence reduces the demand for real balances. The domestic price level consequently rises, and hence the exchange rate has to depreciate to maintain purchasing power parity. The anticipation of depreciation induces an actual depreciation of the domestic currency. Finally, a one time increase in the foreign currency price of tradeable goods, given monetary and fiscal policies, and exchange rate expectations causes the exchange rate to appreciate by the same proportion, so that the domestic currency price of tradeables remains unchanged.

### The Role of Expectations

The equilibrium described in the above section depends on the expected rate of depreciation of the domestic currency. Dornbusch (1976) assumed that the expected rate of change in the exchange rate is a linear function of the current and long-run values of the exchange rate, and then verified that the actual path will satisfy this relation. In this study, this expectational variable is made endogenous by imposing the requirement of rational expectations. As Turnovsky and Kingston (1977) pointed out, any expectational hypothesis is necessarily to some extent arbitrary, but the rational expectations hypothesis does have the important advantage of being the one hypothesis which yields forecasts which are consistent with predictions of the model. And for that reason it is of considerable interest.

$x_t$  in Equation 10 is the expected rate of depreciation of the domestic currency. By first order approximation<sup>1</sup>

$$x_t = E_t(s_{t+1} - s_t) \quad (11)$$

where  $S_{t+1}(S_t)$  is the logarithm of the forward (Spot) exchange rate, and  $E_t [ ]$  denotes the expectations

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<sup>1</sup>See Appendix 2 for explanation.

operator and it is based on the information available to asset holders at that time. Substituting Equation 11 into Equation 8 and solving for  $S_t$  yields

$$s_t = \frac{1}{1+\lambda} [m - a_t - \gamma y_t + \delta r^*_t + \lambda E_t(s_{t+1}) - p^*_t] \quad (12)$$

The current exchange rate is strongly influenced by expectations of its future value and is dependent on information that underlies these expectations. It is important to specify how this expectational variable is determined. Imposing the requirements of rational expectations, we obtain

$$\begin{aligned} E_t(s_{t+1}) &= \frac{1}{1+\lambda} E_t [m_{t+1} - a_{t+1} - \gamma y_{t+1} + \delta r^*_{t+1} \\ &\quad + \lambda E_{t+1}(s_{t+2}) - p^*_{t+1}] \end{aligned} \quad (13)$$

Note that

$$E_t[E_{t+1}(s_{t+2})] = E_t(s_{t+2}) \text{ and also}$$

$$\begin{aligned} E_t(s_{t+2}) &= \frac{1}{1+\lambda} E_t [m_{t+2} - a_{t+2} - \gamma y_{t+2} + \delta r^*_{t+2} \\ &\quad + \lambda E_{t+2}(s_{t+3}) - p^*_{t+2}] \end{aligned} \quad (14)$$

Substituting Equation 14 into 13 and applying the same procedure iteratively to  $E_t(S_{t+3})$  and so on, we obtain the expression

$$E_t(s_{t+1}) = \frac{1}{1+\lambda} \sum_{j=1}^{\infty} E_t[m_{t+j} - a_{t+j} - \gamma y_{t+j} + \delta r_{t+j}^* - p_{t+j}^*] \left(\frac{\lambda}{1+\lambda}\right)^{j-1} \quad (15)$$

provided that  $\lim_{R \rightarrow \infty} E_t(s_{t+R} \left(\frac{\lambda}{1+\lambda}\right)^R) = 0$

meaning that the exchange rate cannot be expected to run off to zero or to infinity. Substituting Equation 15 into Equation 12 yields

$$s_t = \frac{1}{1+\lambda} [m_t - a_t - \gamma y_t + \delta r_t^* - p_t^* + \sum_{j=1}^{\infty} E_t(m_{t+j} - a_{t+j} - \gamma y_{t+j} + \delta r_{t+j}^* - p_{t+j}^*) \left(\frac{\lambda}{1+\lambda}\right)^j] \quad (16)$$

This is the rational expectations solution for the equilibrium exchange rate. We realize that the current equilibrium exchange rate depends on the current monetary and real variables, as well as on the current expectations concerning the future behavior of these exogenous monetary and real variables. Let us also note that our model satisfies the homogeneity postulate. An increase

in the nominal quantity of money that is expected to persist leads to an equiproportionate increase in the equilibrium exchange rate level. This is because money is the only nominal asset; the index bonds are real, not nominal assets.

Examining Equation 15 we recognize that monetary and fiscal policies affect expectations of future exchange rates. This leads to the fact that predictions of future exchange rates based solely on interest-rate-parity conditions can be highly inaccurate, because expected future exchange rates are influenced by expected monetary and fiscal policies as well.

#### Explanation for Exchange Rate Volatility

In the past years, exchange rates have exhibited wider fluctuations than had been expected, and some attention has been devoted to explaining the causes of this short-run volatility. Perspectives on volatility are provided by Schafer (1976), Dornbusch (1976), Kouri (1976) and Branson (1976).

Exchange rate volatility may be attributed to the fact that expectations about future exchange rates are imprecise. Insights into why expectations are imprecise and thus, into why observed exchange rates have been so volatile can be obtained from the model developed in

this section. Rewriting Equation 15 here for convenience,

$$E_t(S_{t+1}) = \frac{\lambda}{1+\lambda} \sum_{j=1}^{\infty} E_t [m_{t+j} - a_{t+j} - \gamma y_{t+j} + \delta r^*_{t+j} - P^*_{t+j}] \left(\frac{\lambda}{1+\lambda}\right)^j$$

We recognize that today's expectations of tomorrow's exchange rate depend on the current expectations of the entire future time paths of both the money supply and all variables that influence money demand. Consequently, to the extent that expectations of these time paths are imprecise and subject to sudden shifts (e.g., when newly available economic data differ from earlier predictions and lead to revised expectations about the money supply path that the central monetary authority will pursue), both exchange rate expectations and observed exchanged rates will also be subject to sudden shifts.

This analysis suggests that changes in expectations about policy variables may be an important cause of exchange rate volatility. Consequently, making information available would allow market participants to predict more accurately the time paths of policy variables, or pursuing smoother time paths of policy variables, should reduce exchange rate volatility to some extent.



Secondly, the analysis suggests that we would expect a positive correlation between the error term (that is the difference between the observed policy variable and the forecast of the policy variable), and the observed exchange rates. This hypothesis can be empirically tested. In order to carry out this, we need to observe that the money supply in a given period is made up of an anticipated component, which the individual predicts based upon all the available information, plus a random unanticipated component, which can be positive, negative, or zero. This represents actual money supply deviations from the expected money supply. Let us approximate the imprecision in the expectations of the time paths of the exogenous policy variables by the money supply deviation from the expected money supply. We would then expect this residual to be positively correlated with the observed exchange rates. This test will be carried out in Chapter 4.

The model developed in this section will be used to examine the effects of anticipated and unanticipated monetary disturbances on the exchange rate level. These results will be empirically tested in the fourth chapter. To achieve this, the money supply has to be forecasted. The predicted values will constitute the anticipated money supply component and the residuals will represent the unanticipated component. These components will be

used to test the theoretical results to be obtained in the next section.

Wealth variables are not completely predetermined. Bonds are perfectly mobile, and hence, any gap in the domestic trade balance must be filled up with bond flows. Hence, domestic net exports plus interest on foreign bonds equals net foreign bond issues to the domestic country. These bond flows in connection with current account surpluses makes the wealth variable endogenous. The analysis will focus on the short-run comparative statics of the system, under two conditions:

- (a) Partial rational expectations, under which agents treat  $A$  as exogenous, i.e.  $E[A_{t+i}] = A_t$  but the actual  $A$  changes according to Equation 7.
- (b) Full rational expectations, under which  $A$  is recognized as endogenous.

Purchasing power parity which we assume obtains continuously, implies that we can use the domestic price level or the exchange rate interchangeably, since the movements in the two variables are the same. If  $A_t$  is treated as exogenous by agents, then the time path of the price level can be obtained from the money market equilibrium condition stated in Equation 11. Note that  $x = P_{t+1} - P_t$  and hence we have

$$\lambda P_{t+1} - (1 + \lambda)P_t = a_t - m_t - e_1 \quad (17)$$

where  $e_1$  represents all the other variables. Solving for  $P_t$  in Equation 17 we obtain

$$P_t = \frac{\sum_0^{\infty} (m_{t+i} - a_{t+i} + e_1) \left(\frac{\lambda}{1+\lambda}\right)^i}{(1 + \lambda)} \quad (18)$$

Note that this solution is the same as in Equation 16.

If  $A_t$  is recognized as endogenous, its time path is then governed by Equation 7, which is written here for convenience:

$$\begin{aligned} \frac{\dot{A}}{r^*} &= Y + A - E\left(Y + A, \frac{M}{P} + \frac{A}{r^*}\right) \\ &= H\left(Y, A, \frac{M}{P}, r^*\right) \end{aligned} \quad \begin{aligned} H_Y &> 0, \quad H_A < 0 \\ H_{\frac{M}{P}} &< 0, \quad H_{r^*} > 0 \end{aligned}$$

An independent increase in real output raises real expenditures, but assuming a marginal propensity to consume of less than unity, part of the increased real output leads to foreign asset accumulation. An increase in  $A$  above its stationary state value leads to foreign asset decumulation, because the increase in  $A$  leads to an increase in real wealth which results in increased expenditures. An increase in real balances raises real

wealth and hence, real expenditures, and this leads to foreign asset decumulation. An independent increase in the foreign real interest rate, induces agents to shift out of money and into bonds, but decreases real wealth, and hence, expenditures. This leads to net accumulation.

Let us define

$B_t$  = Total (private plus government)  
holding of foreign assets

$G_t$  = Government holding of foreign assets

$A_t$  = Private holding of foreign assets

$Z_t$  = The purchase by the Government at time  
 $t$  of real securities.

hence

$$B_t = G_t + A_t$$

$$G_t = G_{t-1} + Z_t$$

At a given time,  $t$ ,  $B_t$  is given, i.e.,

$$\frac{dB_t}{dZ_t} = 0, \text{ but } \frac{dA_t}{dZ_t} = -1.$$

Now let us assume that saving is proportional to any discrepancy between desired and actual wealth, and that desired wealth is solely a function of real output,

and hence, the net accumulation of foreign assets is given by

$$B_{t+1} - B_t = e_o - g[B_t - G_t + M_t - P_t]$$

or

$$Z_{t+1} + A_{t+1} - A_t = ep - gA_t - gM_t + gP_t$$

taking logs on both sides we approximately obtain

$$a_{t+1} - b_o a_t = e_o + gP_t - gm_t - Z_{t+1}; \text{ note } b_o = (1 - g).$$

This is the discrete-time form of Equation 7, and

$$\frac{da_t}{dZ_{t+i}^e} = \begin{matrix} 0 & \text{for } i \neq 0 \\ -1 & \text{for } i = 0 \end{matrix}$$

where  $dZ_{t+i}^e$  is the change (in expectation) of open market purchase. Hence, if  $A_t$  is recognized as endogenous, we have two simultaneous difference equations to solve for the time paths of  $P_t$  and  $A_t$ . The system becomes:

$$\lambda p_{t+1} - (1 + \lambda)p_t - a_t = -e_1 - m_t \quad (19)$$

$$a_{t+1} - b_o a_t - gp_t = e_o - gm_t - Z_{t+1} \quad (20)$$

These two equations, 19 and 20, can be collapsed into one equation to obtain either

$$\alpha_2 P_{t+2} + \alpha_1 P_{t+1} + \alpha_0 P_t = [e_0 - (1 - b_0)e_1] + (b_0 - g)m_t - m_{t+1} - Z_{t+1}$$

or

$$\begin{aligned} \alpha_2 a_{t+2} + \alpha_1 a_{t+1} + \alpha_0 a_t = & -(e_0 + ge_1) + \lambda g(m_t - m_{t+1}) \\ & + (1 + \lambda)Z_{t+1} - \lambda Z_{t+2} \end{aligned}$$

To obtain rational expectations solutions for the price level and the level of foreign assets, conjecture at a solution of the form

$$\begin{aligned} P_t = & \gamma_0 + \gamma_1 \sum_0^{\infty} m_{t+i} r^i + \gamma_2 \sum_0^{\infty} m_{t+i} s^i + \gamma_3 \sum_0^{\infty} Z_{t+1+i} r^i \\ & + \gamma_4 \sum_0^{\infty} Z_{t+1+i} s^i + \gamma_5 \theta_1^t + \gamma_6 \theta_2^t \end{aligned} \quad (21)$$

and

$$\begin{aligned} a_t = & \phi_0 + \phi_1 \sum_0^{\infty} m_{t+i} r^i + \phi_2 \sum_0^{\infty} m_{t+i} s^i + \phi_3 \sum_0^{\infty} Z_{t+1+i} r^i \\ & + \phi_4 \sum_0^{\infty} Z_{t+1+i} s^i + \phi_5 \theta_1^t + \phi_6 \theta_2^t \end{aligned} \quad (22)$$

where  $r$  and  $s$  are the roots obtained from the equation

$$\alpha_2 + \alpha_1 r + \alpha_0 r^2 = 0 \text{ where}$$

$$\alpha_2 = \lambda$$

$$\alpha_1 = -(1 + \lambda + \lambda b_0)$$

$$\alpha_0 = (1 + \lambda)b_0 - g$$

and

$$\gamma_0 = \frac{(1 - b_0)e_1 - e_0}{1 + g - b_0}$$

$$\gamma_1 = \frac{r(b_0 - g) - 1}{D}$$

$$\gamma_2 = \frac{b_0 - g}{(1 + \lambda)b_0 - g} - \gamma_1$$

$$\gamma_3 = \frac{-r}{D}$$

$$\gamma_4 = \frac{r}{D} - \frac{1}{(1 + \lambda)b_0 - g}$$

$$r, s = \frac{-\alpha_1 \pm \sqrt{(\alpha_1)^2 - 4\alpha_0\alpha_2}}{2\alpha_0}$$

$$D = [(1 + \lambda + \lambda b_0)^2 - 4\lambda(b_0 + \lambda b_0 - g)]^{1/2}$$

$$\theta_1 = \frac{1}{r}$$

$$\theta_2 = \frac{1}{s}$$

$$\phi_0 = \frac{ge_1 + e_0}{1 + g - b_0}$$

$$\phi_1 = \gamma_1 \left( \frac{\lambda - (1 + \lambda)r}{r} \right)$$

$$\phi_2 = \frac{\lambda g}{(1 + \lambda)b_0 - g} - \phi_1$$

$$\phi_3 = \gamma_3 \left( \frac{\lambda - (1 + \lambda)r}{r} \right)$$

$$\phi_4 = \frac{1 + \lambda}{(1 + \lambda)b_0 - g} - \phi_3$$

$$\phi_5 = \gamma_5 [\lambda \theta_1 - (1 + \lambda)]$$

$$\phi_6 = \gamma_6 [\lambda \theta_2 - (1 + \lambda)]$$

The determination of these coefficients are fully discussed in Appendix 3.

The problem with this solution is that  $\text{Max}[|r|, |s|] = |r| > 1$  for all  $\lambda$ ,  $b_0$  and  $g_i$  and hence the solution does not converge. To solve this, let us define  $M^*$  such that

$$\sum_0^{\infty} (M_{t+i} - M^*) k^i$$

exists, and we assume converges. Where  $k = \text{max}[|r|, |s|]$

and suppose  $r > 1 > s > 0$ . Let  $V_t = (M_t - M^*)$ , hence

$$\sum_0^{\infty} (M_{t+i} - M^*) r^i = \sum_0^{\infty} V_{t+i} r^i \text{ converges. For example, if}$$

$M_t = M^*$  for all  $t > t$ , then the above converges thus

$$\lambda P_{t+1} - (1 + \lambda)P_t - a_t = -V_t - (e_1 + m^*) \quad (23)$$



$$a_{t+1} - b_0 a_t - gP_t = -gV_t - Z_{t+1} + (e_0 - gm^*) \quad (24)$$

Then for the solution, we obtain

$$\begin{aligned} P_t = & \gamma_0 + \gamma_1 \sum_0^{\infty} V_{t+i} r^i + \gamma_2 \sum_0^{\infty} V_{t+i} s^i + \gamma_3 \sum_0^{\infty} Z_{t+1+i} r^i \\ & + \gamma_4 \sum_0^{\infty} Z_{t+1+i} s^i + \gamma_5 \theta^t_1 + \gamma_6 \theta^t_2 \end{aligned} \quad (25)$$

$$\begin{aligned} a_t = & \phi_0 + \phi_1 \sum_0^{\infty} V_{t+i} r^i + \phi_2 \sum_0^{\infty} V_{t+i} s^i + \phi_3 \sum_0^{\infty} Z_{t+1+i} r^i \\ & + \phi_4 \sum_0^{\infty} Z_{t+1+i} s^i + \phi_5 \theta^t_1 + \phi_6 \theta^t_2 \end{aligned} \quad (26)$$

where in Equation 25,

$$\hat{\gamma}_0 = m^* - \frac{(1-b_0)e_1 - e_1}{1 + g - b_0}$$

The values of  $\gamma_1$  through  $\gamma_4$  and  $\phi_0$  through  $\phi_4$  are the same as given before.<sup>1,2</sup>

Then for  $r > 1 > s$ ,  $|\theta_2| > 1$  and convergence requires  $\gamma_6 = \phi_6 = 0$ . The solution obtained in Equations 25 and 26 gives the time paths of the price level  $P_t$  and the level

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<sup>1</sup>The relationship between  $\phi_5$  and  $\gamma_5$  is derived in Appendix 4.

<sup>2</sup>If  $\frac{M_{t+1}}{M_t}$  is a constant, the model could be extended to the case where there is a steady state positive money growth rate.

of foreign assets  $a_I$ , for  $I \geq t$  given expectations at  $t$ , and the level of foreign assets  $a_t$  at time  $t$ . If expectations are fulfilled, it will be the actual solution. Suppose at some  $I \geq t$ , expectations are revised, then  $V_{t+i}$ , and  $Z_{t+i}$  are revised. The new solution must be convergent and must satisfy the initial condition, i.e.,  $a_I$  is unchanged but  $P_I$  jumps. Hence, we must recalculate Equations 25 and 26 for the new  $V$  and  $Z$  and choose  $\gamma_5$  and  $\phi_5$  such that the initial condition for  $a_I$  is satisfied.

From the two roots of the quadratic equation  $\alpha_2 + \alpha_1 r + \alpha_0 r^2 = 0$ , we have  $r + s = \frac{\alpha_1}{\alpha_0}$  and  $rs = \frac{\alpha_2}{\alpha_0}$ . This implies that if  $r > 1 > s > 0$ , then  $\gamma_0 = (1 + \lambda)b_0 - g > 0$  and therefore we can establish the following a priori restrictions.

$$\gamma_1 < 0, \gamma_2 > 0, \gamma_3 < 0, \gamma_4 > 0$$

$$\phi_1 > 0, \phi_2 > 0, \phi_3 > 0, \phi_4 > 0$$

It is also possible that  $r < -1$  and  $0 < s < 1$  in which case  $\alpha_0 = (1 + \lambda)b_0 - g < 0$ . This implies that  $b < \frac{1}{2 + \lambda}$  or  $g = 1 - b > \frac{1 + \lambda}{2 + \lambda}$ . And hence, the greater the interest semielasticity of demand for money,  $\lambda$ , the greater the marginal propensity to consume out of wealth,  $g$ . Since the homogenous solution is  $\theta_1^t = r^{-t}$ , if  $r < -1$  this implies oscillatory behavior of prices and foreign assets.

If  $r < -1$ , and  $0 < s < 1$ , then our a priori restrictions on the parameters will be as follows:

$$\gamma_1, \gamma_2, \gamma_3, \gamma_4 > 0$$

$$\phi_1 < 0, \phi_2 > 0, \phi_3 < 0 \text{ and } \phi_4 > 0$$

For the homogenous solution in both cases (when  $r > 1$ , and  $r < -1$ ) we have,

$$\gamma_5 = \frac{\phi_5}{\lambda \theta_1 - (1 + \lambda)} = \frac{-\phi_5 r}{(1 + \lambda)r - \lambda}$$

Therefore  $\text{sign}(\gamma_5) = -\text{sign}(\phi_5)$ .

Now suppose, at  $t = 0$ , expectations are revised either due to an immediate (unanticipated) change in the money supply  $M$  or in government bond holdings,  $G$ , or due to an anticipated change in  $M$  or  $G$ , we will obtain

$$da_0 = -dZ_0 = -d\phi_5 + \left[ \sum_0^{\infty} (\phi_1 r^i + \phi_2 s^i) dV_1^e + \sum_0^{\infty} (\phi_3 r^i + \phi_4 s^i) dZ_{1+i}^e \right]$$

$$d\phi_5 = -\left[ dZ_0 + \sum_0^{\infty} (\phi_1 r^i + \phi_2 s^i) dV_1^e + \sum_0^{\infty} (\phi_3 r^i + \phi_4 s^i) dZ_{1+i}^e \right]$$

$$\text{and } \gamma_5 = \frac{\phi_5 r}{\lambda - r(1 + \lambda)} \text{ implies } d\gamma_5 = \left[ \frac{-r}{(1 + \lambda)r - \lambda} \right] d\phi_5$$

Hence given  $\theta_1^t = r^{-t}$ ; and  $da_0 = -dZ_0$ , then for  $da_t$ , for  $t > 0$  we have

$$\begin{aligned}
da_t &= \theta_1^t d\phi_5 + [\overset{\infty}{\Sigma}_0(\phi_1 r^i + \phi_2 s^i)dv^e_{t+i} + \overset{\infty}{\Sigma}_0(\phi_3 r^i + \phi_4 s^i)dz^e_{t+1+i}] \\
&= -dz_0 \theta_1^t - \theta_1^t [\overset{\infty}{\Sigma}_0(\phi_1 r^i + \phi_2 s^i)dv^e_i + \overset{\infty}{\Sigma}_0(\phi_3 r^i + \phi_4 s^i)dz^e_{1+i}] \\
&\quad + [\overset{\infty}{\Sigma}_0(\phi_1 r^i + \phi_2 s^i)dv^e_{t+i} + \overset{\infty}{\Sigma}_0(\phi_3 r^i + \phi_4 s^i)dz^e_{t+1+i}]
\end{aligned}$$

and hence,

$$\begin{aligned}
da_t &= -dz_0 \theta_1^t - \theta_1^t [\overset{t-1}{\Sigma}_0(\phi_1 r^i + \phi_2 s^i)dv^e_i + \overset{t-1}{\Sigma}_0(\phi_3 r^i + \phi_4 s^i)dz^e_{1+i}] \\
&\quad - \theta_1^t [\overset{\infty}{\Sigma}_t(\phi_1 r^i + \phi_2 s^i)dv^e_i + \overset{\infty}{\Sigma}_t(\phi_3 r^i + \phi_4 s^i)dz^e_{1+i}] \\
&\quad + [\overset{\infty}{\Sigma}_0(\phi_1 r^i + \phi_2 s^i)dv^e_{t+i} + \overset{\infty}{\Sigma}_0(\phi_3 r^i + \phi_4 s^i)dz^e_{t+1+i}] \\
&= -dz_0 \theta_1^t - \theta_1^t [\overset{t-1}{\Sigma}_0(\phi_1 r^i + \phi_2 s^i)dv^e_i \\
&\quad + \overset{t-1}{\Sigma}_0(\phi_3 r^i + \phi_4 s^i)dz^e_{1+i}] \\
&\quad - \theta_1^t [\overset{\infty}{\Sigma}_0 \phi_1 r^{t+i} + \phi_2 s^{t+i})dv^e_{t+i} \\
&\quad + \overset{\infty}{\Sigma}_0(\phi_3 r^{t+i} + \phi_4 s^{t+i})dz^e_{t+1+i}] \\
&\quad + [\overset{\infty}{\Sigma}_0(\phi_1 r^i + \phi_2 s^i)dv^e_{t+i} + \overset{\infty}{\Sigma}_0(\phi_3 r^i + \phi_4 s^i)dz^e_{t+1+i}]
\end{aligned}$$

Therefore,

$$\begin{aligned}
 da_t &= -dz_0 \theta_1^t - \theta_1^t \left[ \sum_0^{t-1} (\phi_1 r^i + \phi_2 s^i) dv^e_i \right. \\
 &\quad + \sum_0^{t-1} (\phi_3 r^i + \phi_4 s^i) dz^e_{1+i} \left. \right] \\
 &\quad + \left[ \sum_0^\infty \{ (\phi_1 r^i + \phi_2 s^i) - \theta_1^t (\phi_1 r^{t+i} + \phi_2 s^{t+i}) \} dv^e_{t+i} \right. \\
 &\quad \left. + \left[ \sum_0^\infty \{ (\phi_3 r^i + \phi_4 s^i) - \theta_1^t (\phi_3 r^{t+i} + \phi_4 s^{t+i}) \} dz^e_{t+i+1} \right] \right]
 \end{aligned}$$

Further simplification leads to

$$\begin{aligned}
 da_t &= -dz_0 \theta_1^t - \theta_1^t \left[ \sum_0^{t-1} (\phi_1 r^i + \phi_2 s^i) dv^e_i \right. \\
 &\quad + \sum_0^{t-1} (\phi_3 r^i + \phi_4 s^i) dz^e_{1+i} \left. \right] \\
 &\quad + \sum_0^\infty \left[ \phi_2 s^i \left\{ \frac{\phi_1}{\phi_2} \left( \frac{r^i}{s^i} \right) + 1 - \theta_1^t \left( \frac{\phi_1}{\phi_2} \left( \frac{r^{t+i}}{s^i} \right) + s^t \right) \right\} dv^e_{t+i} \right. \\
 &\quad \left. + \sum_0^\infty \left[ \phi_4 s^i \left\{ \frac{\phi_3}{\phi_4} \left( \frac{r^i}{s^i} \right) + 1 - \theta_1^t \left( \frac{\phi_3}{\phi_4} \left( \frac{r^{t+i}}{s^i} \right) + s^t \right) \right\} dz^e_{t+i+1} \right] \right]
 \end{aligned}$$

Note that  $\theta_1^t = r^{-t}$ , hence we have

$$\begin{aligned}
da_t &= -dz_0 \theta_1^t - \theta_1^t \left[ \sum_0^{t-1} (\phi_1 r^i + \phi_2 s^i) dv_i^e \right. \\
&\quad \left. + \sum_0^{t-1} (\phi_3 r^i + \phi_4 s^i) dz_{1+i}^e \right] \\
&\quad + \sum_0^{\infty} [\phi_2 s^i (1 - (\frac{s}{r})^t) dv_{t+i}^e + \phi_4 s^i (1 - (\frac{s}{r})^t) dz_{t+i+1}^e] \quad (26a)
\end{aligned}$$

This is the change in  $a_t$  (for  $t \neq 0$ ) due to a change in expectations (or unanticipated policy) at  $t = 0$ .

Similarly, we have

$$\begin{aligned}
dP_t &= d\gamma_0 + \left[ \sum_0^{\infty} \{ (\gamma_1 r^i + \gamma_2 s^i) dv_{t+i}^e + (\gamma_3 r^i + \gamma_4 s^i) dz_{t+1+i}^e \} \right] \\
&\quad + \theta_1^t d\gamma_5
\end{aligned}$$

Note that

$$d\gamma_5 = \left[ \frac{-r}{(1+\lambda)r - \lambda} \right] d\phi_5 \text{ and hence we have}$$

$$\begin{aligned}
dP_t &= d\gamma_0 + \sum_0^{\infty} [(\gamma_1 r^i + \gamma_2 s^i) dv_{t+i}^e + (\gamma_3 r^i + \gamma_4 s^i) dz_{t+1+i}^e] \\
&\quad + \theta_1^t \left[ \frac{r}{(1+\lambda)r - \lambda} \right] [dz_0 + \sum_0^{\infty} \{ (\phi_1 r^i + \phi_2 s^i) dv_i^e \\
&\quad + (\phi_3 r^i + \phi_4 s^i) dz_{i+1}^e \}]
\end{aligned}$$

Further simplification leads to

$$\begin{aligned}
 dP_t = & d\gamma_0 + \left[ \frac{rdz_0}{(1+\lambda)r - \lambda} \right] \theta_1^t + \left[ \sum_0^{t-1} (\phi_1 r^i + \phi_2 s^i) dv^e_i \right. \\
 & + \left. \sum_0^{t-1} (\phi_3 r^i + \phi_4 s^i) dz^e_{i+1} \right] \left[ \frac{r\theta_1^t}{(1+\lambda)r - \lambda} \right] \\
 & + \left( \sum_0^\infty [(\phi_1 r^{t+i} + \phi_2 s^{t+i}) dv^e_{t+i} \right. \\
 & + \left. (\phi_3 r^{t+i} + \phi_4 s^{t+i}) dz^e_{t+i+1}] \right) \left[ \frac{r\theta_1^t}{(1+\lambda)r - \lambda} \right] \\
 & + \sum_0^\infty [(\gamma_1 r^i + \gamma_2 s^i) dv^e_{t+i} + (\gamma_3 r^i + \gamma_4 s^i) dz^e_{t+1+i}]
 \end{aligned}$$

and hence

$$\begin{aligned}
 dP_t = & d\gamma_0 + \left[ \frac{rdz_0}{(1+\lambda)r - \lambda} \right] \theta_1^t + \left( \left[ \sum_0^{t-1} (\phi_1 r^i + \phi_2 s^i) dv^e_i \right. \right. \\
 & + \left. \left. \sum_0^{t-1} (\phi_3 r^i + \phi_4 s^i) dz^e_{i+1} \right] \right) \left[ \frac{r\theta_1^t}{(1+\lambda)r - \lambda} \right] \\
 & + \sum_0^\infty [(\gamma_1 r^i + \gamma_2 s^i) + \left( \frac{\phi_1 r^{1+i}}{(1+\lambda)r - \lambda} + \left( \frac{\phi_2 r \left(\frac{s}{r}\right)^t}{(1+\lambda)r - \lambda} \right) s^i \right)] dv^e_{t+i} \\
 & + \sum_0^\infty [(\gamma_3 r^i + \gamma_4 s^i) + \left( \frac{\phi_3 r^{1+i}}{(1+\lambda)r - \lambda} + \left( \frac{\phi_4 r \left(\frac{s}{r}\right)^t}{(1+\lambda)r - \lambda} \right) s^i \right)] dz^e_{t+1+i}
 \end{aligned}$$

grouping the terms together leads to

$$\begin{aligned}
dP_t &= dy_0 + \left[ \frac{rdz_0}{(1+\lambda)r - \lambda} \right] \theta_1^t + \left[ \sum_0^{t-1} (\phi_1 r^i + \phi_2 s^i) dv^e_i \right. \\
&\quad + \sum_0^{t-1} (\phi_3 r^i + \phi_4 s^i) dz^e_{i+1} \left. \right] \left[ \frac{r\theta_1^t}{(1+\lambda)r - \lambda} \right] \\
&\quad + \sum_0^\infty \left[ \left( \frac{\phi_1 r}{(1+\lambda)r - \lambda} + \gamma_1 \right) r^i + \left( \gamma_2 + \frac{\phi_2 r \left(\frac{s}{r}\right)^t}{(1+\lambda)r - \lambda} \right) s^i \right] dv^e_{t+i} \\
&\quad + \sum_0^\infty \left[ \left( \frac{\phi_3 r}{(1+\lambda)r - \lambda} + \gamma_3 \right) r^i + \left( \gamma_4 + \frac{\phi_4 r \left(\frac{s}{r}\right)^t}{(1+\lambda)r - \lambda} \right) s^i \right] dz^e_{t+1+i}
\end{aligned}$$

Note that  $\frac{\phi_3}{\gamma_3} = - \left[ \frac{(1+\lambda)r - \lambda}{r} \right]$

and  $\frac{\phi_1}{\gamma_1} = - \left[ \frac{(1+\lambda)r - \lambda}{r} \right]$

and hence,

$$\frac{\phi_1 r}{(1+\lambda)r - \lambda} + \gamma_1 = 0$$

and

$$\frac{\phi_3 r}{(1+\lambda)r - \lambda} + \gamma_3 = 0$$



Thus:

$$\begin{aligned}
 dP_t = & dm^* + \theta^t \left[ \frac{rdz_0}{(1+\lambda)r - \lambda} \right] + \left[ \sum_0^{t-1} (\phi_1 r^i + \phi_2 s^i) dv^e_i \right. \\
 & + \sum_0^{t-1} (\phi_3 r^i + \phi_4 s^i) dz^e_{i+1} \left. \right] \left[ \frac{r\theta^t}{(1+\lambda)r - \lambda} \right] \\
 & + \sum_0^\infty \left[ \left( \gamma_2 + \frac{\phi_2 r \left(\frac{s}{r}\right)^t}{(1+\lambda)r - \lambda} \right) s^i \right] dv^e_{t+i} \\
 & + \left( \gamma_4 + \frac{\phi_4 r \left(\frac{s}{r}\right)^t}{(1+\lambda)r - \lambda} \right) s^i dz^e_{t+1+i}
 \end{aligned} \tag{26b}$$

This is the change in  $P_t$  due to change in expectations at  $t = 0$  (or policy at  $t = 0$ ). Also note that for  $t = 0$ ,

$\sum_{i=0}^{t-1} = 0$ . Equations 26a and 26b will be used to analyze

the effects of monetary policies under rational expectations.

The above model is similar to the model of Dornbusch and Fischer (1980) in some respects. However, their study emphasizes the relationship between the behavior of the exchange rate and the current account. In particular, they look at how the current account through its effects on net asset positions, and therefore on assets markets, determines the path of the exchange rate over time. The purpose of this study is to examine the effects of anticipated and unanticipated monetary disturbances on the exchange rate and to test the derived results.

The main concern in subsequent sections is to analyze both the short run and the steady state effects of various types of monetary disturbances. The short run effects will be related to the steady state effects, and hence it is convenient to focus first on the latter.

### Steady State

The steady state of the system is attained when all accumulation ceases and the expected rate of exchange depreciation is zero. Balance of trade will be zero and demand for money will be equal to the supply of money. For either monetary policy, this is described by the set of equations:

$$\frac{M^d}{P} = \theta(Y, r^*) \left( \frac{M}{P} + \frac{A}{r^*} \right) = \frac{M}{P} \quad (27)$$

$$\frac{\dot{A}}{r^*} = Y - E\left(Y, \frac{M}{P} + \frac{A}{r^*}\right) = 0 \quad (28)$$

Equation 27 is the money market equilibrium condition, and Equation 28 is the current account equation, which asserts that in the steady state equilibrium, real income and real expenditures,  $E$ , are equalized, and hence foreign asset accumulation equals zero. The variables  $A$  and  $M/P$  are endogenous in the long run, and are determined by Equations 27 and 28. By taking the total

differentials of the two equilibrium conditions, 27 and 28 we obtain the following system of equations:

$$W[\theta_y dY + \theta_{r^*} dr^*] + \theta d\left(\frac{M}{P}\right) + \theta d\left(\frac{A}{r^*}\right) - d\left(\frac{M}{P}\right) = 0$$

$$dy) - E_y dY - E_w d\left(\frac{M}{P}\right) - E_w d\left(\frac{A}{r^*}\right) = 0$$

For given levels of  $Y$  and  $r^*$

$$\begin{bmatrix} \theta - 1 & \frac{\theta}{r^*} \\ -E_w & \frac{-E_w}{r^*} \end{bmatrix} \begin{bmatrix} d\left(\frac{M}{P}\right) \\ dA \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and hence  $d\left(\frac{M}{P}\right) = 0$ ,  $dA = 0$  in the steady state. Consequently, a policy of pure monetary expansion leads to an equiproportionate increase in the price level in the long run (and short run). A 1% increase in the nominal money supply will lead to a 1% increase in the price level and hence a 1% increase in the exchange rate. Real income, and real wealth will remain unchanged, as will the domestic interest rate.

In the steady state, private security holdings are unchanged,  $dA = 0$ . An open market operation which is an increase in the money supply brought about by purchasing privately held securities results in an overshooting of

the exchange rate, and hence a reduction in real wealth. This leads to balance of trade surplus and therefore an accumulation of foreign assets. As the foreign assets are accumulated the exchange rate falls; with  $A$  increasing, and with the exchange rate falling the trade surplus decreases. The process continues until the initial real equilibrium is reattained. At that point the exchange rate will have depreciated, since the nominal money supply is higher while all real variables are unchanged. The cumulative effect of the monetary expansion is thus clearly an equiproportionate depreciation, but only across steady states.

The effect of an independent increase in  $r^*$  can be obtained from the following system of equations:

$$W\theta_{r^*}dr^* + \theta d\left(\frac{M}{P}\right) + \frac{\theta}{r^*} dA - \frac{\theta A}{r^{*2}} dr^* - d\left(\frac{M}{P}\right) = 0$$

$$-E_w(d\left(\frac{M}{P}\right)) - \frac{E_w}{r^*} dA + \frac{E_w A}{r^{*2}} dr^* = 0$$

and hence

$$\begin{bmatrix} \theta - 1 & \frac{\theta}{r^*} \\ -E_w & \frac{-E_w}{r^*} \end{bmatrix} \begin{bmatrix} d\left(\frac{M}{P}\right) \\ dA \end{bmatrix} = \begin{bmatrix} \left(\frac{\theta A}{r^{*2}} - W\theta_{r^*}\right) dr^* \\ -\frac{E_w A}{r^{*2}} dr^* \end{bmatrix}$$

$$\frac{d\left(\frac{M}{P}\right)}{dr^*} = \frac{E_W W \theta_{r^*} / r^*}{-\frac{E_W}{r^*} (\theta - 1) + \frac{\theta}{r^*} E_W} = W \theta_{r^*} < 0$$

and

$$\begin{aligned} \frac{dA}{dr^*} &= \frac{\frac{E_W A}{r^{*2}} - E_W W \theta_{r^*}}{-\frac{E_W}{r^*} (\theta - 1) + \frac{\theta}{r^*} E_W} \\ &= \left( \frac{A}{r^{*2}} - W \theta_{r^*} \right) r^* > 0 \end{aligned}$$

An independent increase in the foreign interest rate reduces the equilibrium real money stock both because it lowers the preferred share of real balances in wealth and because it reduces the level of real wealth. With real balances comprising a smaller fraction (and hence with real assets comprising a larger fraction) of wealth, we must have an increase in the equilibrium price level and thus a depreciation of the exchange rate.

### Short Run Analysis

In this model, it is possible to introduce a distinction between anticipated changes in exogenous variables based on previously available information, and unanticipated changes in exogenous variables due to the receipt of new information. The analysis of the anticipated changes is

an attempt to explain the impact of an increase in the money supply, for example, when the increase is announced prior to the actual increase. This type of analysis is of interest because it shows how a lag in the implementation of a policy affects the economy before and after the policy is carried out. This analysis is done following the procedure originally outlined by Fischer (1979).

Two forms of government monetary policies are considered and compared:

- (a) Open market operation: an operation in which the central bank increases money supply by buying government bonds from the private individuals.
- (b) Pure monetary expansion: this involves injection of new money into the system. This happens when budget deficits are covered by money issue.

The initial effect of the open market purchase leaves the total wealth unaffected, and hence has no direct wealth effect on expenditures for real commodities. This operation creates an excess supply of money and an equivalent excess demand for bonds. For any particular means of increasing the money supply, the short run effects of the policy change depend critically on the response of expectations. Given our assumption of perfect capital

mobility, i.e., interest rate parity, our interest rate can be low when appreciation is anticipated or high if there is an expectation of depreciation. Thus, the variable  $x_t$  plays the same role as the domestic interest rate in a conventional short run closed economy model. Monetary expansion, on the other hand, does not involve the removal of financial assets from the private sector but instead entails the injection of new money into the system, and thus increases the net wealth holdings of the private individuals.

Specifically, we shall consider the following monetary disturbances under the two forms of monetary policies specified above:

(a) Unanticipated Transitory Disturbance: Its effect on the exchange rate will be represented as  $\frac{ds_t}{dm_t} \Big|^{TU}$ . The superscript indicates the type of monetary disturbance.

(b) Unanticipated Permanent Disturbance: represented

by  $\frac{ds_t}{dm_t} \Big|^{PU}$

(c) Anticipated Transitory Disturbance:  $\frac{ds_t}{dm_t} \Big|^{TA}$

(d) Anticipated Permanent Disturbance:  $\frac{ds_t}{dm_t} \Big|^{PA}$

In the analyses that follow, subscripts will be put on the above differentials to indicate the type of monetary policy: omo for open market operation and ME for pure monetary expansion by injection of new money. The purpose of these analyses is to explore the similarities and differences between open market operation and the issue of new money under flexible exchange rates.

#### Unanticipated transitory monetary disturbance

In this section we investigate the impact of an increase in only the current period's money supply. Consider first the case in which the monetary authorities suddenly decide to increase the money supply by an open-market operation. It is also assumed that in subsequent periods the authorities revert to the usual level of money supply and to a policy of balancing the budget, so that no further additions to the money supply take place. Under an open market policy, the central bank increases the money supply by buying bonds at the going market price from private individuals, and hence leaves the private nominal wealth unaffected, so that

$$dM + \frac{P}{r^*} dA = 0$$



$$\begin{aligned}\frac{dA}{A} &= \frac{-r^*}{P} \frac{dM}{A} \\ &= \frac{-dM}{M} \cdot \frac{M}{P} \frac{r^*}{A}\end{aligned}$$

Using Equation 5, we obtain

$$\frac{dA}{A} = \frac{-dM}{M} \left( \frac{\theta}{1-\theta} \right)$$

and hence,

$$dm_t - da_t = dm_t \left( \frac{1}{1-\theta} \right)$$

The effect of an unanticipated transitory open market purchase of bonds on the equilibrium exchange rate is obtained by totally differentiating Equation 16 and setting  $dy_t = dp_t^* = dr_t^* = 0$  and  $dm_t - da_t = dm_t \left( \frac{1}{1-\theta} \right)$  and also  $dm_{t+j} = dy_{t+j} = dr_{t+j}^* = dp_{t+j}^* = 0$  and  $da_{t+j} = 0$  for  $j = 1, 2, \dots$ , i.e., since it is known to be transitory, we can assume the accumulation behavior does not change. Hence,

$$\left. \frac{ds_t}{dm_t} \right|_{\text{omo}}^{\text{TU}} = \frac{1}{(1+\lambda)(1-\theta)} > 0$$

At the initial value of  $S_t$  and  $P_t$ , domestic asset holders find themselves with an excess supply of money and excess demand for bonds. At a point in time, the stock of  $A$  is given and hence balance of trade flows cannot affect  $A$ . As they attempt to buy foreign assets they drive up the exchange rate. Hence, the initial effect of the increase in the money stock is a depreciation in the exchange rate, to maintain asset market equilibrium. The increase in the price level due to exchange rate depreciation reduces real wealth. If agents expect the open market operation to be purely transitory, its change will not affect real expenditures and wealth accumulation by the private sector. They will then anticipate the exchange rate to appreciate in the period after the disturbance. This lowers the interest rate, and hence leads to an increase in real balance demand and a decrease in the demand for real bonds to establish equilibrium. The depreciation in the exchange rate on impact depends on the initial proportions of assets in portfolios,  $\theta$ , and the elasticity of demand for money with respect to the interest rate  $\lambda$ . The larger the  $\lambda$ , and the smaller the  $\theta$ , the smaller the impact on the exchange rate. After the corrective monetary policy, the system will return to the original equilibrium. This is shown graphically below.

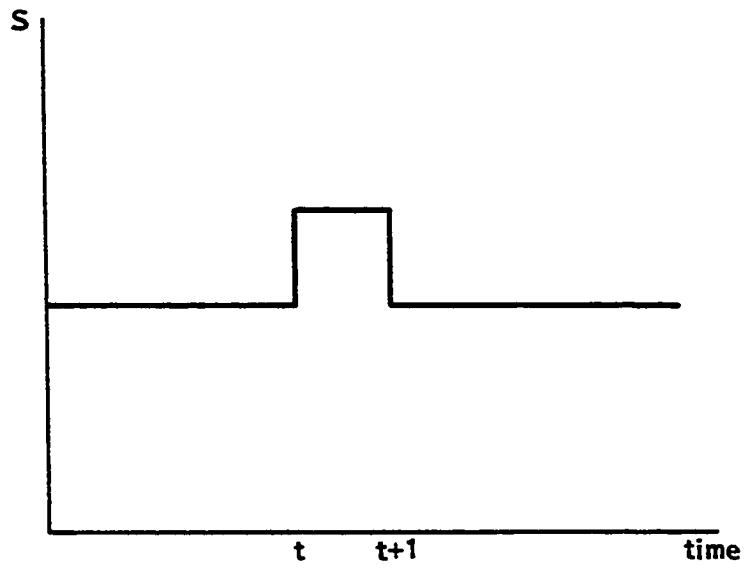


Figure 1. The time path of the exchange rate when a transitory change in the money supply is unanticipated

The effect of unanticipated transitory monetary expansion by means of injection of new money into the system produces similar results, the exchange rate depreciates. As the public's holdings of money increase, they attempt to rebalance portfolios by buying foreign assets. With given supplies of this asset, the increased demand causes the exchange rate to depreciate. The effects of injection of new money is derived by totally differentiating Equation 16 and setting  $dy_{t+j} = dr^*_{t+j} = dp^*_{t+j} = 0$  for  $j = 0, 1, 2 \dots$  and also  $da_{t+j} = 0$ , meaning that the monetary expansion is known to be transitory, and we can assume the accumulation behavior does not change. Hence

$$\left. \frac{ds_t}{dm_t} \right|_{\substack{TU \\ ME}} = \frac{1}{1 + \lambda} < 1$$

Since  $\frac{1}{1+\lambda} < 1$ , unanticipated monetary expansion causes the current exchange rate to depreciate, but less than in proportion to the increase in the money stock. In contrast to the open market operation, the issue of new money which is not accompanied by an equivalent removal of financial assets from the private sector, does increase the net asset holdings of the private sector. Thus, the issue of new money has a wealth

effect while the open market operation has no such effect. After the initial adjustment, the stock of real balances increases. To the extent that agents expect the monetary expansion to be purely transitory implies that expenditures and real wealth decumulation is unlikely to be affected by the short run disturbances. Agents also expect the exchange rate to be revalued, and this tends to make the short run rate behave similarly, thereby providing an offsetting effect.

In the above analysis, the endogenous wealth changes and its long run implications were ignored. To take these into consideration we need to examine the analysis under full rational expectations, where endogenous changes in foreign assets are considered. However, the unanticipated transitory monetary expansion could be ignored under full rational expectations, since presumably a transitory effect would be ignored by agents.

The magnitudes of the short run adjustments in the exchange rate for the two forms of monetary expansion are different and are interesting to compare. We can deduce that in either case, exchange rate depreciation occurs, but the extent of depreciation will be greater for the open market operation than for the case when new money is issued. An explanation runs as follows. Open market purchases can be regarded as the difference

between monetary expansion and debt expansion, with the changes in government expenditures cancelling out. Hence, the effects of an open market operation must be qualitatively the same as that of monetary expansion, but reinforced by the additional effect of debt retirement. Hence, we obtain a stronger decline in asset yields under open market operation.

#### Unanticipated permanent monetary disturbance

Permanent unanticipated monetary disturbance is an increase in the money supply in one period that is not retracted in subsequent periods. The effects of the two forms of the monetary expansion in the steady state are as described in an earlier section. There is full neutrality in the steady state.

Under partial rational expectations, when  $A$  is treated as exogenous, the effect of an unanticipated monetary expansion by means of injection of new money into the system is derived by totally differentiating Equation 16 and setting  $dy_{t+j} = dp^*_{t+j} = dr^*_{t+j} = da_{t+j} = 0$  for  $j = 0, 1, 2, \dots$ . Hence

$$\begin{aligned} ds_t &= \frac{1}{1+\lambda} \left[ dm_t + \sum_1^{\infty} dE_t(m_{t+j}) \left( \frac{\lambda}{1+\lambda} \right)^j \right] \\ &= \frac{1}{1+\lambda} \cdot \left[ 1 + \sum_1^{\infty} \left( \frac{\lambda}{1+\lambda} \right)^j \right] dm_t \end{aligned}$$

Therefore

$$\left. \frac{ds_t}{dm_t} \right|_{\substack{\text{PU} \\ \text{ME}}} = 1$$

An unanticipated permanent increase in the stock of money supply causes the exchange rate to depreciate proportionally and immediately. There is therefore, no accompanying process of accumulation. Long run neutrality result holds in this case, since there are no other nominal assets in this model;  $A_t$  is a real asset, not a nominal asset. Changes in the nominal money supplies have no long run effects on real variables, such as consumption and trade balances, but rather an equiproportionate long run effects on exchange rates and price levels. This is the same result obtained in the steady state analysis in an earlier section.

Under full rational expectations, when  $A$  is recognized as endogenous, the effect of a monetary expansion is as derived below. The unanticipated monetary disturbance does not affect the level of foreign assets at the time of implementation of the policy; also the current money supply level jumps up to the new steady state money supply level, since the disturbance is permanent, hence

$$dG_0 = dZ_0 = 0 \text{ and } dZ_j^e = 0 \text{ for } j > 0 \text{ and } dM - dM_0 \text{ and } dV_i = 0$$

for all  $i$ . And hence from Equation 26b we have

$$dp_t = d\gamma_0 = dm_0$$

The unanticipated permanent monetary expansion which moves the current steady state money supply level to its new steady state level implies that

$$\left. \frac{dP_t}{dm_t} \right|_{\substack{\text{PU} \\ \text{ME}}} = 1$$

Hence a pure unanticipated monetary expansion which is expected to persist under full rational expectations also leads to full neutrality. The effect of the unanticipated permanent monetary expansion is shown in Figure 2.

Under partial rational expectations, permanent unanticipated change in  $M$  and  $G$  (at  $t = 0$ ) is described by totally

differentiating Equation 16, and setting  $dy_{t+j} = dp^*_{t+j} = dr^*_{t+j} = 0$  for all  $j$  and  $dm_{t+j} - da_{t+j} = dm_t \left(\frac{1}{1+\lambda}\right)$ , hence

$$ds_t = \frac{1}{1+\lambda} [dm_t - da_t + \sum_{j=1}^{\infty} (dm_{t+j} - da_{t+j}) \left(\frac{\lambda}{1+\theta}\right)^j]$$

$$ds_t = \frac{1}{1+\lambda} \left[ \frac{1}{1-\theta} dm_t + \sum_{j=1}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^j \left(\frac{1}{1-\theta}\right) dm_t \right]$$

$$\left. \frac{ds_t}{dm_t} \right|_{\text{omo}} \begin{array}{l} \text{PU} \\ \text{omo} \end{array} = \frac{1}{1+\lambda} \left[ \frac{1}{1+\theta} + \lambda \left(\frac{1}{1-\theta}\right) \right]$$

$$= \frac{1}{1-\theta} > 1$$



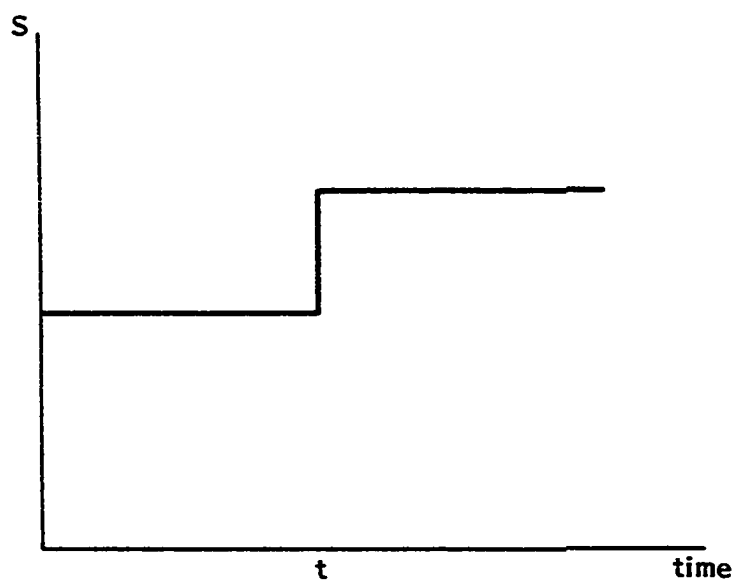


Figure 2. The time path of the exchange rate when a permanent change in the money stock is unanticipated

This is the impact effect, assuming agents believe  $da_{t+j} = da_{t+i}$ . This result shows that the short run exchange rate depreciates more than in proportion to the increase in the money stock. The extent of the depreciation depends on the initial proportions of assets in portfolios. The larger is the proportion of assets held in the form of nominal balances,  $\theta$ , the greater is the extent of depreciation. This reflects the fact that larger shares of domestic currency assets in domestic portfolios, imply larger changes in the foreign currency valuations of portfolios following an unanticipated depreciation of domestic currency. Consequently, larger excess demands for foreign assets are induced by the depreciation. This result is similar to that obtained by Dornbusch (1976). In Dornbusch's model, the occurrence of this phenomenon is associated with the regressivity of expectations, and the differential speeds of adjustment between the money market and the goods market. The present analysis shows that it is the wealth effect in the demand for money function which is the important factor in this phenomenon, and not the assumption of regressive expectations.

The above analysis has considered the stock of foreign assets  $A$ , as constant over time, that is changes in  $A$  are exogenous. However, for a permanent change in the money

supply by open market operation, the balance of trade flows can affect the stock of A over time and hence, should be taken into account. Since the exchange rate depreciates, the value of real money balances falls, and hence the real wealth declines. From the accumulation equation

$$\dot{\frac{A}{r^*}} = Y + A - E\left(Y + A, \frac{M}{P} + \frac{A}{r^*}\right)$$

we observe that the reduction in real wealth leads to balance of trade surplus and therefore an accumulation of foreign assets. With the accumulation of foreign assets, the real wealth level goes up and leads to an increase in the demand for real balances, and hence an exchange rate appreciation. The above analysis can be modified to incorporate endogenous wealth changes, by examining Equations 26a and 26b. A permanent unanticipated change in M and G (at  $t = 0$ ) means  $dG_0 = dZ_0$ ;  $dM = dM_0$  and  $d\gamma_0 = dm_0$ ,  $dZ_j^e = 0$  for  $j > 0$ ; and  $dV_i = 0$  for all  $i$  hence

$$da_t = -\theta_1^t dZ_0$$

and

$$dp_t = dm_0 + \left[\frac{r}{(1+\lambda)r - \lambda}\right] \theta_1^t dZ_0$$

Since  $dZ_0 = \left(\frac{\theta}{1-\theta}\right) dm_0$ , we have

$$\left. \frac{dP_t}{dm_t} \right|_{\text{omo}}^{\text{PU}} = 1 + \theta^t \left[ \frac{r}{(1+\lambda)r - \lambda} \right] \left( \frac{\theta}{1-\theta} \right)$$

Hence, unanticipated open market operation under full rational expectations causes the exchange rate to depreciate more than in proportion to the increase in the money stock, when  $r > 1$ , and when  $r < -1$ , it oscillates. The magnitude of the short run adjustments in the exchange rate for open market operation under partial and full rational expectations are different and interesting to compare. Under partial rational expectations, when  $A_t$  is treated as exogenous, the effect of the open market operation is

$$\left. \frac{dP_0}{d_{\text{omo}}} \right|_{\text{A exog.}} = \frac{1}{1-\theta} = 1 + \frac{\theta}{1-\theta}$$

and under full rational expectations, we have

$$\left. \frac{dP_0}{d_{\text{omo}}} \right|_{\text{FRE}} = 1 + \left[ \frac{r}{(1+\lambda)r - \lambda} \right] \left[ \frac{\theta}{1-\theta} \right]$$

and since  $0 < \frac{r}{(1+\lambda)r - \lambda} < 1$ , we conclude that

$$\left. \frac{dp_0}{d_{omo}} \right|_{FRE} < \left. \frac{dp_0}{dm_0} \right|_{A \text{ Exog}}$$

That is, even though there is overshooting in each case, there is less overshooting under full rational expectations. This reflects the fact that following an unanticipated depreciation, real wealth declines, and hence a reduction in real expenditures. Consequently, the trade surplus induced by the depreciation and the accompanying foreign asset accumulation provides an offsetting anticipation of exchange rate appreciation under full rational expectations.

The change in  $a_t$ , the level of foreign assets is described by

$$da_t = -\theta^t {}_1 dz_0$$

if  $r > 1$ , then the time path of  $a_t$  approaches  $\phi_0$ , the steady state level, from below. Otherwise if  $r < -1$ ,  $a_t$  converges towards the steady state level in an oscillatory manner. The time paths are described in Figure 3 below. This result shows that after the monetary policy, foreign assets are accumulated. This continues with  $A$  rising until the current account is back in balance. At that point the rate of accumulation of the foreign assets

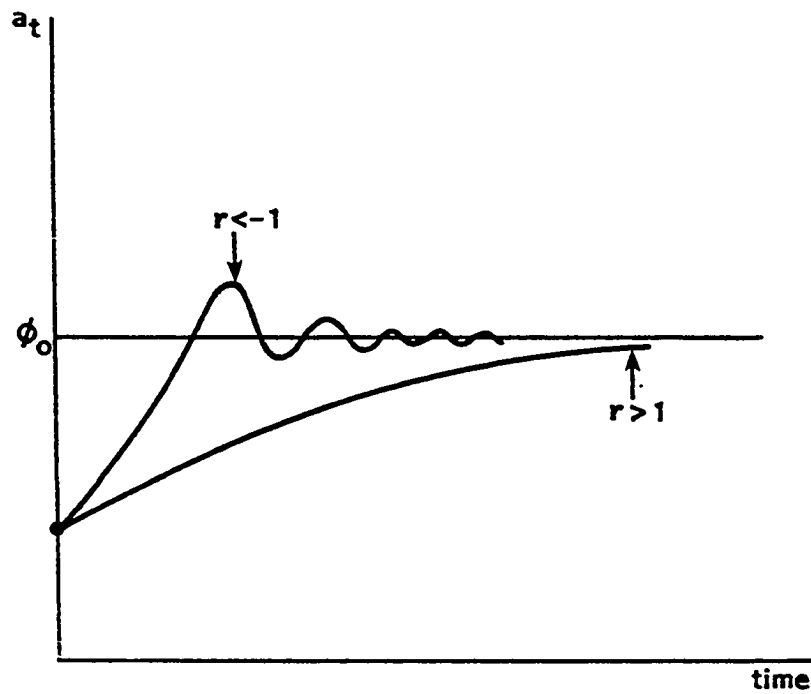


Figure 3. The time path of  $a_t$  due to a permanent open market operation policy

is back to zero, so that the system is back in steady state equilibrium.

The change in the price level  $p_t$  due to a policy at  $t = 0$  is described by

$$dp_t = dm_0 + \left( \frac{r}{(1+\lambda)r - \lambda} \right) \theta^t dz_0$$

When the marginal propensity to consume out of wealth,  $g$ , is small then  $\alpha_0 = (1+\lambda)bo-g > 0$  and hence  $r > 1$ , and  $0 < s < 1$ . Under that condition, the price level approaches its steady state value  $p^*$  geometrically from above. And when the marginal propensity to consume out of wealth,  $g$ , is large then  $\alpha_0 = (1+\lambda)bo-g < 0$  and hence  $r < -1$ , and  $0 < s < 1$ . Under that condition the price level approaches its steady state value in an oscillatory fashion. This is described in Figure 4.

The unanticipated discrete monetary expansion causes an immediate exchange rate depreciation. The magnitude of this depreciation is that necessary to establish short run equilibrium. Thereafter, in the absence of additional monetary disturbances, the system will converge towards the new steady state. As the foreign assets are accumulated (due to the fact that real wealth has fallen), the trade surplus decreases due to rising wealth. This continues with  $A$  rising until the current account is back in balance. At that point the rate of accumulation of net foreign assets is back to zero, so that the system is back in steady state equilibrium. Throughout the adjustment process there is a surplus in the current

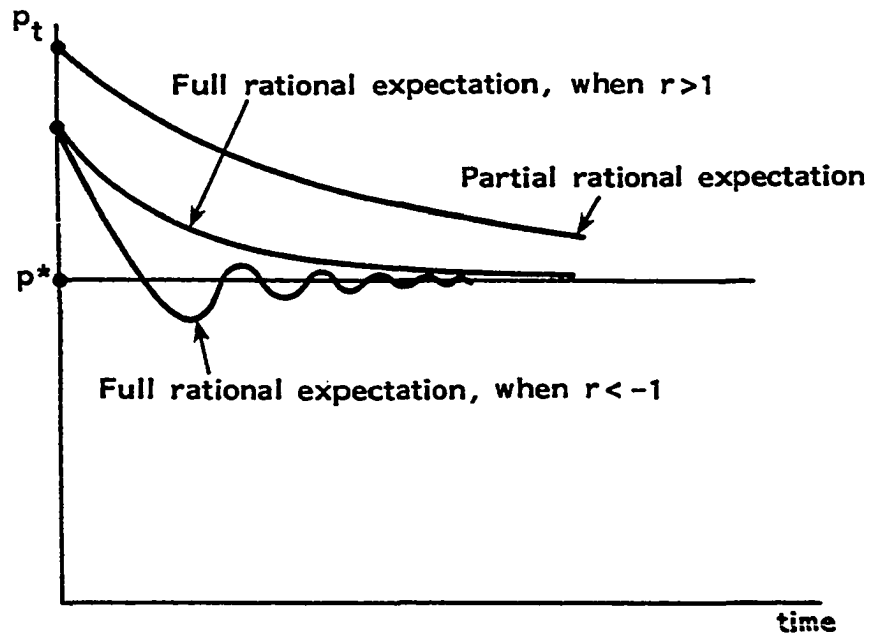


Figure 4. The time path of  $p_t$  due to a permanent open market operation policy



account. The process continues until the initial real equilibrium is reattained. At that point the exchange rate will have depreciated since the nominal money supply is higher while all real variables are unchanged. The cumulative effect of the monetary expansion is thus clearly an equiproportionate depreciation, but only across steady states. Figure 5 below shows the time path of the exchange rate under various expectations assumptions. At time  $t = 0$  when the monetary policy takes place the spot rate depreciates by the magnitude  $\frac{1}{1-\theta}$ , under partial rational expectations. This is represented by AB. If we neglect subsequent wealth changes, that is, if A is exogenous and unchanging, then the exchange rate stays at that level and follows the path BC. However, if agents treat A as exogenous, that is  $E[A_{t+1}] = A_t$  but the actual A changes according to Equation 7, then the exchange rate appreciates and follows the path BE, and approaches the new steady state value. Under full rational expectations, the initial impact is the depreciation represented by AD, and thereafter as foreign assets are accumulated, the exchange rate appreciates and converges towards its new steady state equilibrium. The exchange rate appreciates geometrically at the rate  $\theta_1^t$  if  $r > 1$ , and in an oscillatory fashion if  $r < -1$ .

We also realize that the extent of depreciation will be greater for an open market operation than for the

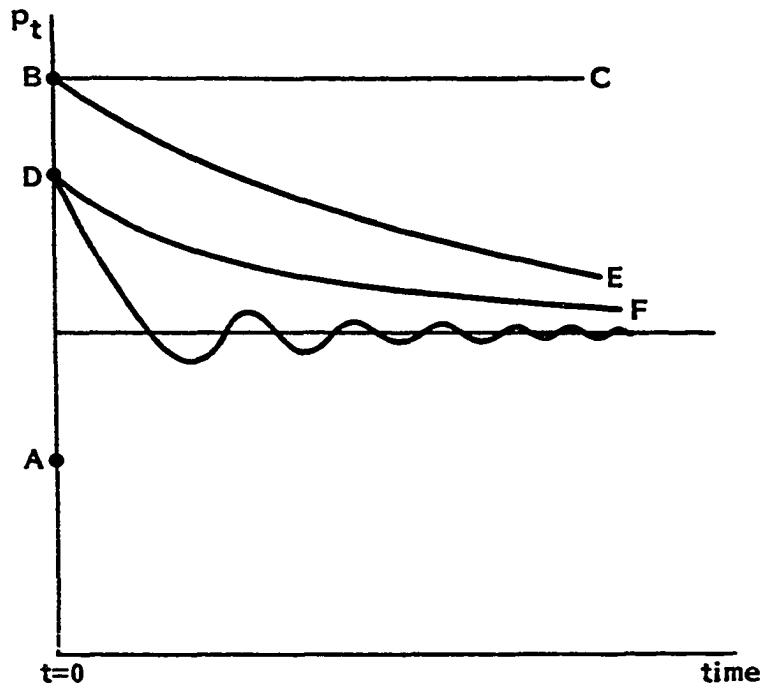


Figure 5. The time path of the exchange rate when a permanent change in the money stock by means of open market operation is unanticipated

case when new money is issued, the explanation being that the open market operation is reinforced by the additional effect of the debt retirement. Hence we obtain a stronger decline in asset yields under open market operation.

Comparing the results under permanent disturbance to the case when the monetary disturbance is transitory, we realize that the extent of depreciation in the case of permanent disturbance is greater. This is because a change in the money supply which is supposed to persist affects the current and all future money supplies, and hence must have a greater impact on the exchange rate than a transitory change which affects only the current money supply.

#### Anticipated disturbances

Assume that the initial position of the economy just before the policy change is a steady state equilibrium, and the government is initially inactive, that is all parameters of government policy are set at zero. Therefore if the change in the money stock at period  $t + T$  is entirely unanticipated, and that up to that time the money stock has been constant, then the economy will be in steady state equilibrium up to that time, and when the monetary expansion occurs, the exchange rate

depreciation occurs. In contrast, suppose we are examining an increase in the nominal money supply at time  $t + T$  in the future and suppose that this increase in the money supply is announced at time  $t$ , in an economy that is initially in steady state equilibrium. If portfolio holders anticipate that on the day the money supply increases the price level and the exchange rate would jump up, this expectation will immediately drive portfolio holders out of money and into securities. Hence, the impact effect of the announcement is to generate a jump in the price level and exchange rate depreciation. From then on, the exchange rate will continue to depreciate. These results operate through the effects of the anticipated change in the money stock on absorption. The anticipated increase in the money stock causes an anticipated and therefore, actual depreciation of the currency. The anticipation of depreciation lowers the real balances and hence, real wealth, and spending. From Equation 7, this gives rise to a current account surplus. The process continues up to the time the money supply increases, with all the jumps in the exchange rate and the price level having been anticipated initially. The current account surplus leads to accumulation of foreign assets, and leads to an increase in real wealth which raises demand for real balances which implies a moderation

of the price rise and the exchange rate depreciation. This offsetting effect of the current account surplus on the exchange rate depreciation will mean a smaller total increase in the exchange rate by the time  $t + T$  as compared to the effect of the monetary expansion, under partial rational expectations, when agents treat  $A$  as exogenous and constant. After the money stock is actually increased, real wealth increases; this now leads to a deficit and decumulation of foreign assets until the initial real equilibrium is reattained. Throughout the adjustment process the exchange rate is depreciating (this will be discussed in detail later on). At time  $t + T$  the exchange rate cannot be expected to jump, but nominal money does increase; hence, from then on agents would expect a decline in the exchange rate depreciation, which means a moderation in the rate of increase in the exchange rate. In order to increase demand for the larger stock of money, the rate of interest must fall, and this occurs due to the expected appreciation of the exchange rate, after time  $t + \tau$ . From then on, depreciation continually decreases until it reaches zero in the steady state. However, if agents treat  $A$  as exogenous and constant, then there will not be any offsetting effect of the current account surplus on the exchange rate depreciation.

Anticipated permanent monetary disturbance

Under partial rational expectations, a permanent anticipated open market operation at  $t = \tau$ ; that is at  $t = 0$ , agents believe there will be a permanent change at  $t = \tau$ . It follows from Equation 18 that

$$\begin{aligned} dm_0 - da_0 &= 0, & t + i < \tau \\ &= dm\left(\frac{1}{1-\theta}\right), & t + i \geq \tau \end{aligned}$$

From Equation 18, we have

$$dp_t = \frac{1}{1+\lambda} \sum_{i=0}^{\infty} (dm_{t+i} - da_{t+i}) \left(\frac{\lambda}{1+\lambda}\right)^i$$

For  $t \leq \tau$ , we have

$$\begin{aligned} dm_{t+i} - da_{t+i} &\neq 0 && \text{when } t + i \geq \tau \\ &&& \text{or for } i \geq \tau - t \end{aligned}$$

Therefore the time path of the exchange rate due to a change in expectations at  $t = 0$  is given by

$$dS_t = \left(\frac{1}{1-\theta}\right) \left(\frac{1}{1+\lambda}\right) \sum_{i=\tau-t}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^i dm \quad (29)$$

Therefore at  $t = 0$ , the impact effect of the change in expectations is given by  $dS_0 = \left(\frac{1}{1-\theta}\right) \left(\frac{\lambda}{1+\lambda}\right)^\tau dm$

Hence, the impact effect of the announcement is to generate a depreciation of exchange rate. The results suggest that the longer the time lag between the time of the announcement and the time the change actually occurs, the smaller is the initial depreciation of the exchange rate. From Equation 29 above, it is clear that the effects of the anticipated increase in the money stock on the exchange rate are greatest in the period in which the change occurs and are proportional to that change in earlier periods, with the weights declining geometrically. It is plausible to have the increase in the exchange rate level in all earlier periods since the information on which individual's expectations are based changes with the announcement of the monetary policy. At  $t = \tau$ ,  $d(m_{t+i} - a_{t+i}) \neq 0$  for all  $i$ , hence

$$\begin{aligned} dS_{\tau} &= \left(\frac{1}{1-\theta}\right) \left(\frac{1}{1+\lambda}\right) \sum_{i=0}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^i dm_{\tau} \\ &= \left(\frac{1}{1-\theta}\right) dm_{\tau} \end{aligned}$$

The result shows that the exchange rate depreciates proportionately more than the increase in the money supply, at  $t = \tau$ . If agents treat  $A$  as exogenous and fixed, then the exchange rate level stays at the level it realizes at  $t = \tau$ , that is above the full neutrality

level. However, if agents treat  $A$  as exogenous, that is  $e[A_{t+i}] = A_t$  but the actual  $A_t$  changes according to Equation 7, then the exchange rate appreciates at  $t \geq \tau$  and approaches the full neutrality level as  $t \rightarrow \infty$ . This is illustrated in Figure 6.

Comparing the impact effect of unanticipated open market operation with anticipated open market operation impact effect, we realize that

$$\left. \frac{dS_o}{d_{omo}} \right|_{\text{anticipated}} = \left( \frac{\lambda}{1+\lambda} \right)^\tau \left. \frac{dS_o}{d_{omo}} \right|_{\text{unanticipated}}$$

and hence the impact effect of the unanticipated open market operation is a multiple of the impact effect of the anticipated open market operation. And since

$$\left( \frac{\lambda}{1+\lambda} \right)^\tau < 1 \text{ it implies that for } \tau > 0$$

$$\left. \frac{dS_o}{d_{omo}} \right|_{\text{unanticipated}} > \left. \frac{dS_o}{d_{omo}} \right|_{\text{anticipated}}$$

Under full rational expectations, a permanent anticipated open market operation at  $t = \tau$  means that



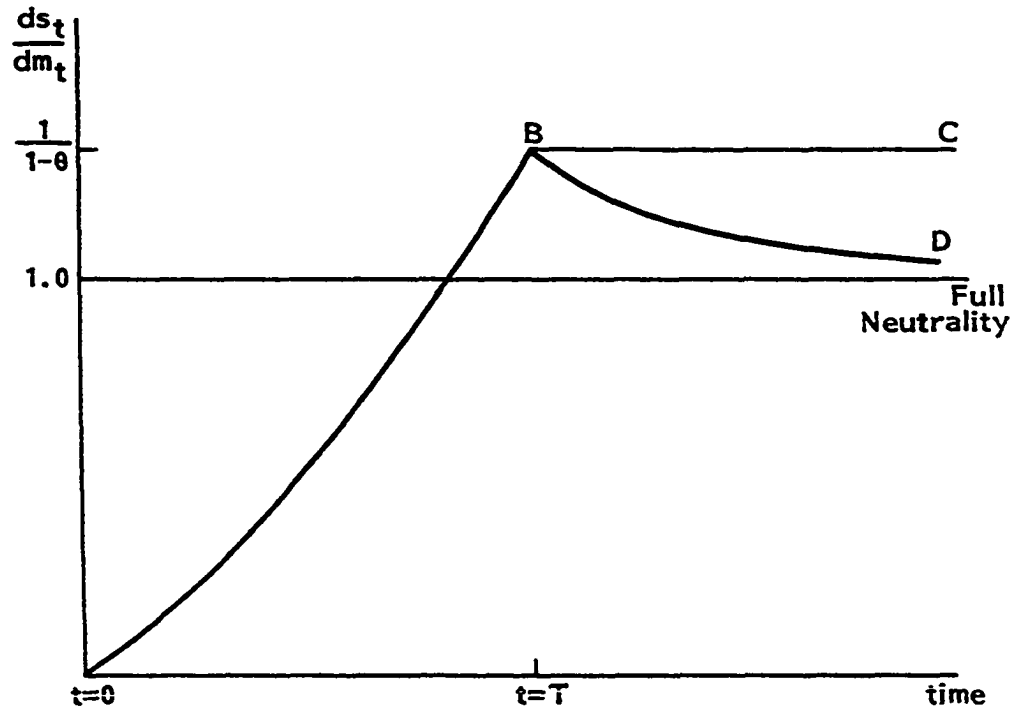


Figure 6. The time path of the exchange rate under partial rational expectations when a permanent open market operation is anticipated

$$dZ_0 = 0$$

$$\begin{aligned} dm_t &= dm^*, \quad t \geq \tau \\ &= 0, \quad t < \tau \end{aligned}$$

which implies that

$$\begin{aligned} dv_i &= -dm^*, \quad i < \tau \\ &= 0, \quad i \geq \tau \end{aligned}$$

and also

$$\begin{aligned} dZ_i &= 0, \quad i \neq \tau \\ dz_t &= dz = \frac{\theta}{1-\theta} dm \end{aligned}$$

Consider first the path of the economy at  $t \geq \tau$ . Hence, from equation 26a we have

$$\begin{aligned} da_\tau &= \theta^\tau \left[ \sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 s^i) \right] dm - dz \left[ \frac{\phi_3}{r} + \frac{\phi_4}{s} \left( \frac{s}{r} \right)^\tau \right] \\ &= \theta^\tau \left[ \sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 s^i) \right] dm - (\phi_3 r^{\tau-1} + \phi_4 s^{\tau-1}) dz \\ &= \phi_5 \phi_1^\tau \end{aligned}$$

where

$$\phi_5 = \left[ \sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 s^i) dm - (\phi_3 r^{\tau-1} + \phi_4 s^{\tau-1}) dz \right]$$

If  $\phi_5 > 0$ , and  $\gamma_5 < 0$ , then  $a_\tau$  will be above its steady state level and consequently lead to decumulation of external assets when  $t > \tau$ . If  $\phi_5 < 0$  and  $\gamma_5 > 0$ , then  $a_\tau$  will be below its steady state level and consequently lead to accumulation of external assets when  $t > \tau$ .<sup>1</sup>

For  $t > \tau$ , the change in  $a_t$  is given by

$$\begin{aligned} da_t &= \theta_1^t dm \left[ \sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 s^i) \right] - \theta_1^t dz [\phi_3 r^{\tau-1} + \phi_4 s^{\tau-1}] \\ &= \phi_1 \theta_1^t \end{aligned}$$

and as  $t \rightarrow \infty$ ,  $\theta_1^t \rightarrow 0$  and therefore  $da_t \rightarrow 0$ . This means that after the monetary policy is implemented, the system will converge towards its steady state value, either from above or from below, depending on the signs of  $\phi_5$  and  $\gamma_5$ . The process continues until the initial

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<sup>1</sup>See Appendix 4 for explanations.

real equilibrium is reattained. The time path of the external assets level towards equilibrium in the long run will be oscillatory for the case when  $r < -1$ .

The time path of  $a_t$  for  $t < \tau$  is as follows:

(i) when  $t = 0$ ,  $da_0 = 0$

(ii) when  $0 < t < \tau$ ,

$$\begin{aligned} da &= \theta^t {}_1 \left[ \sum_{i=0}^{t-1} (\phi_1 r^i + \phi_2 s^i) \right] dm - \left[ \sum_{i=0}^{\tau-t} \phi_2 s^i \left(1 - \left(\frac{s}{r}\right)^t\right) \right] dm \\ &\quad + \left[ \phi_4 s^{\tau-t-1} \left(1 - \left(\frac{s}{r}\right)^t\right) \right] dz \\ &= \theta^t {}_1 \left[ \sum_{i=0}^{t-1} (\phi_1 r^i + \phi_2 s^i) - \sum_{i=0}^{\tau-t} \phi_2 s^i (r^t - s^t) \right] dm \\ &\quad + \left[ \phi_4 s^{\tau-t-1} \left(1 - \left(\frac{s}{r}\right)^t\right) \right] dz \end{aligned}$$

An expected monetary expansion at  $t = \tau$  would cause an immediate depreciation in the domestic currency and hence, a decline in real balances, real wealth, and real expenditures. This would consequently lead to current account surplus. Hence, we would expect an accumulation of foreign assets. This is reinforced by the last term, the expected seizure of external assets by the government from the private individuals, hence we can conclude that  $da_t > 0$ . This means that at  $t = 0$  when expectations are revised due to an anticipated change in the monetary

policy, there is no immediate impact on the level of external assets that is  $a_t$  remains unchanged. But when the anticipated depreciation leads to actual depreciation, we achieve a current account surplus and hence an accumulation of foreign assets. As  $t$  approaches  $\tau$ , we accumulate more and more foreign assets. This process continues until the monetary policy is carried out at  $t = \tau$ . The time path of the foreign asset level in relation to the time path of the exchange rate will be fully discussed later on in this section.

Let us now consider the time path of the price level. From Equation 26b, the time path of the price level due to the announcement at  $t = 0$  is described below:

(i) When  $t = \tau$

$$\begin{aligned} dp_\tau &= dm + \left[ \frac{r\theta_1^t}{(1+\lambda)r - \lambda} \right] [\phi_3 r^{\tau-1} + \phi_4 s^{\tau-1}] dZ - dm \sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 s^i) \\ &= dm + \frac{\phi_5 r\theta_1^\tau}{(1+\lambda)r - \lambda} \end{aligned}$$

If  $\phi_5 < 0$ , it implies that  $-\phi_5 > 0$ . This means that for levels of external assets below the steady state level at  $t = \tau$ , the exchange rate is above its steady state level. Assets are accumulated as  $a_t$  moves up towards its steady state level, at the same time as the exchange rate appreciates. Since asset accumulation is brought about by

a current account surplus, we have a current account surplus accompanying an appreciating exchange rate in the adjustment process. Similarly if  $\phi_5 > 0$ , it implies that  $-\phi_5 < 0$ , and hence for levels of external assets above the steady level at  $t = \tau$ , the exchange rate is below (undershoots) its steady state level. Hence, in that case assets are decumulated as  $a_t$  moves down towards its steady state level, at the same time as the exchange rate depreciates.

(ii) when  $t > \tau$

$$dp_t = dm - \frac{\phi_5 r \theta_1^t}{(1+\lambda)r -}$$

As  $t \rightarrow \infty$ ,  $\theta_1^t \rightarrow 0$  and  $dp_t \rightarrow dm$ . The exchange rate either overshoots or undershoots its long run target at  $t = \tau$  depending on the signs of  $\phi_5$  and  $\phi_5$  and at  $t > \tau$ , it either appreciates or depreciates towards the steady state equilibrium, exponentially if  $r > 1$  or oscillatory fashion if  $r < -1$ .

(iii) when  $t \leq \tau - 1$

$$\begin{aligned}
dp_t = dm - & \left[ \frac{r\theta^t}{(1+\lambda)r - \lambda} \right] \left[ \sum_{i=0}^{t-1} (\phi_1 r^i + \phi_2 s^i) dm \right] \\
& - \sum_{i=0}^{\tau-t} \left[ \gamma_2 + \frac{\phi_2 r \left(\frac{s}{r}\right)^t}{(1+\lambda)r - \lambda} \right] s^i dm \\
& + \left[ \gamma_4 + \frac{\phi_4 r \left(\frac{s}{r}\right)^t}{(1+\lambda)r - \lambda} \right] s^{\tau-t-1} dz
\end{aligned}$$

Hence, when  $t = 0$

$$\begin{aligned}
dp_t = dm - & \sum_{i=0}^{\tau-1} \left[ \gamma_2 + \frac{\phi_2 r}{(1+\lambda)r - \lambda} \right] s^i dm \\
& + \left[ \gamma_4 + \frac{\phi_4 r}{(1+\lambda)r - \lambda} \right] s^{\tau-1} dz \\
= dm - & \sum_0^{\tau-1} \left[ \frac{b_0 - g}{\alpha_0} + \frac{\lambda g}{\alpha_0 ((1+\lambda)r - \lambda)} \right] s^i dm + \frac{1}{\alpha_0} \left[ \frac{\lambda}{r(1+\lambda) - \lambda} \right] s^{\tau-1} dz
\end{aligned}$$

Hence, at  $t = 0$ , when expectations are revised,  $a_t$  is unchanged; the immediate response is a depreciation of the exchange rate. The anticipated increase in the money stock causes an anticipated and therefore an actual depreciation of the domestic currency. Under perfect foresight the magnitude of depreciation due to the announcement generates a movement of the exchange rate in an exponential manner, until the monetary policy is implemented. During this period, real balances, wealth and spending are lowered, and consequently lead to a current account surplus and foreign assets are

accumulated. If agents do not form rational expectations, this implies an appreciating exchange rate which the agents persistently ignore. Under full rational expectations, this is not possible and hence, the initial decline in the stock of real balances is less, implying that the initial depreciation is also less. When the monetary expansion is anticipated, the expected inflation that the anticipated monetary increase induces causes bond stock to rise, through trade balance surplus, and thus exerts a deflationary effect on the price level. Hence, the depreciation of the exchange rate by the time of the policy implementation will be much less than the effect under partial rational expectations. At  $t = \tau$  the monetary authorities carry out the open market operations, a monetary expansion with a contraction in external assets. If  $\phi_5 > 0$ , then  $a_\tau$  is above its steady state level and hence the exchange rate undershoots its steady state target. After the monetary policy is implemented, assets are decumulated, at the same time as the exchange rate continues to depreciate. This process continues until the initial real equilibrium is reattained. The time paths of the exchange rate and the foreign assets are described in Figures 7a and 7b. When the change in the money stock is anticipated, the impact effect of the announcement is to generate exchange rate depreciation represented by AB in Figure 7a. Due to expectations, the exchange



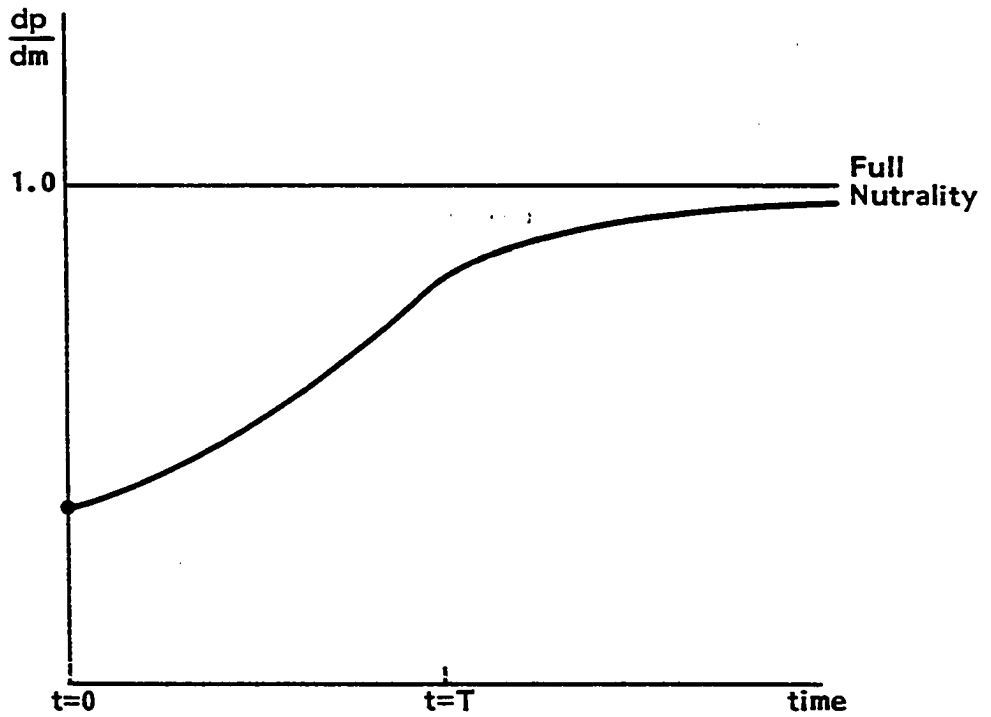


Figure 7a. The time path of the price level (exchange rate) under full rational expectations when an open market operation is anticipated

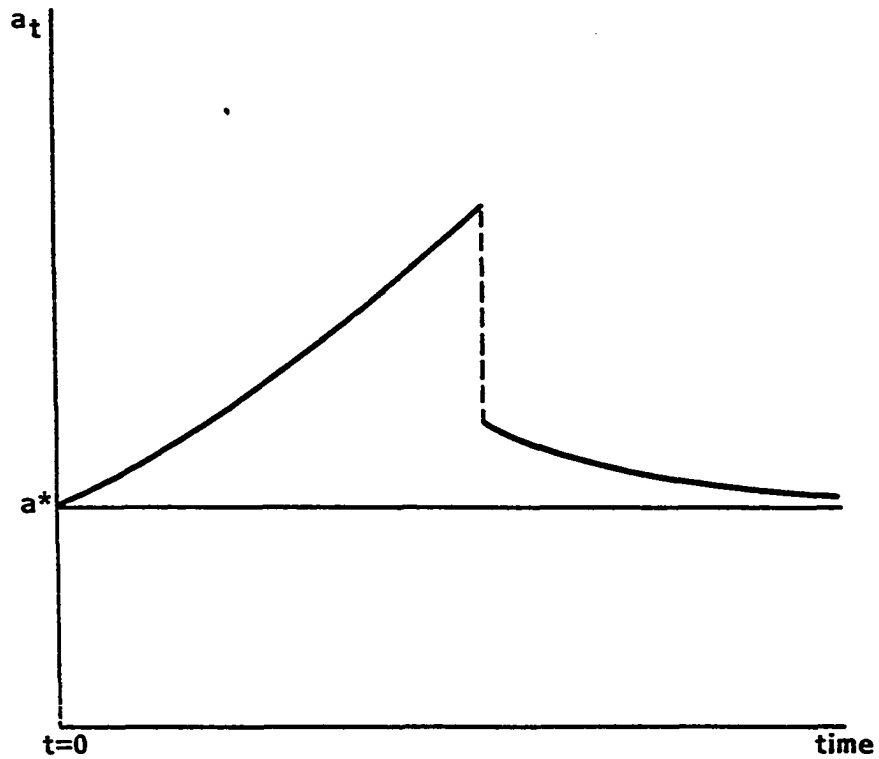


Figure 7b. The time path of the foreign asset level  $a_t$ , under full rational expectations when an open market operation is anticipated

rate continues to depreciate, following the path BC, accompanied by current account surplus and hence external assets accumulation (Figure 7b). After the monetary policy, the exchange rate cannot be expected to depreciate, but nominal money does increase, accompanied by a contraction in external assets. In order to increase demand for the larger stock of money, the rate of interest must fall, and this occurs due to an expected appreciation in the exchange rate. This expected appreciation, therefore, means a moderation in the exchange rate depreciation. Hence at  $t > \tau$  the exchange rate depreciates at a decreasing rate until it reaches zero in the steady state. Using the following assumed parameter values, we can simulate the time path of the price level for illustration:

$g = 0.2$ ,  $b_0 = 0.8$ ;  $\tau = 5$ ;  $\lambda = 0.2$ ;  $\theta = 0.01$ ;  $m^* = 100$ ;  
 $e_0 = 2$ , and  $e_1 = 2$ . Hence, the calculated values of the remaining parameters are  $r = 1.63$ ;  $s = 0.16$ ;  $\alpha_2 = 0.2$ ;  
 $\alpha_1 = -1.36$ ;  $\alpha_0 = 0.76$ ;  $\gamma_0 = 96$ ;  $\gamma_1 = 0.02$ ;  $\gamma_2 = 0.81$ ;  
 $\gamma_3 = -1.47$ ;  $\gamma_4 = 0.15$ ;  $\phi_0 = 6$ ;  $\phi_1 = 0.022$ ;  $\phi_2 = 0.033$ ;  
 $\phi_3 = 1.58$ ;  $\phi_4 = 0.001$ ;  $D = 1.11$ ;  $\theta_1 = 0.61$ . Let us also assume that the money supply is expected to rise from 100 to 200. The calculated exchange rate levels under partial and full rational expectations are shown in Table 1a below.

Table 1a. Calculated exchange rate levels in response to anticipated open market operations

Time	$\frac{dS_t}{dm_t}$ Full rational expectations	$\frac{dS_t}{dm_t}$ Partial rational expectations
Preannouncement	0.0000	0.0000
t = 0	0.0002	0.00023
t = 1	0.0012	0.0008
t = 2	0.0025	0.0047
t = 3	0.0170	0.0281
t = 4	0.0280	0.1684
t = 5	0.9762	1.0100
t = 6	0.9854	1.0100
t = 7	0.9910	1.0100
t = 8	0.9945	1.0100
t = 9	0.9966	1.0100
t = 10	0.9979	1.0100
t $\rightarrow \infty$	$P_t \rightarrow 1.0000$	

The exchange rate undershoots under full rational expectations and overshoots under partial rational expectations at  $t = \tau$ . Under full rational expectations, the exchange rate continues to depreciate at  $t > \tau$ , but at a decreasing rate and is accompanied by external asset decumulation until initial real equilibrium is reattained. Under partial rational expectations when  $a_t$  is treated as exogenous and fixed, the exchange rate does not appreciate after  $t = \tau$ , rather it stays at that level permanently.

However, if  $\phi_5 < 0$ , then  $a_\tau$  is below its steady state level and hence, the exchange rate overshoots its steady state target. After the monetary policy is implemented, assets are accumulated, at the same time as the exchange rate appreciates. This process continues until the initial real equilibrium is reattained. The time paths of the exchange rate and the foreign assets are described in Figures 8a and 8b. When the change in the money stock is anticipated, the impact effect of the announcement is to generate exchange rate depreciation represented by AB in Figure 8a. Due to expectations, the exchange rate continues to rise, following the path BC, accompanied by current account surplus (Figure 8b), and hence foreign asset accumulation. After the monetary policy, the exchange rate cannot be expected to depreciate, but nominal money does increase, hence from then on agents

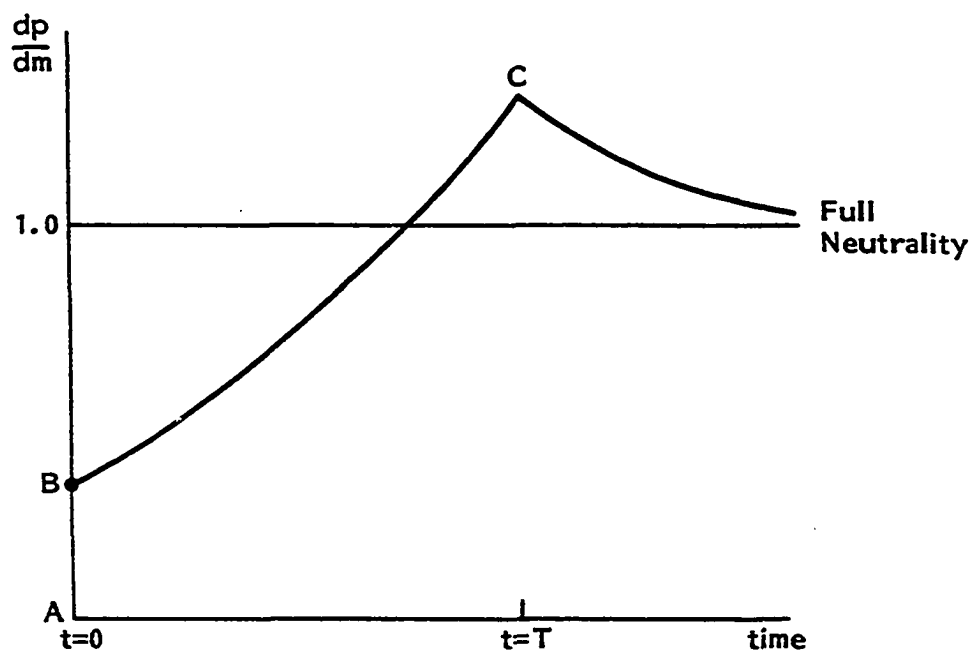


Figure 8a. The time path of the price level (exchange rate) under full rational expectations when an open market operation is anticipated

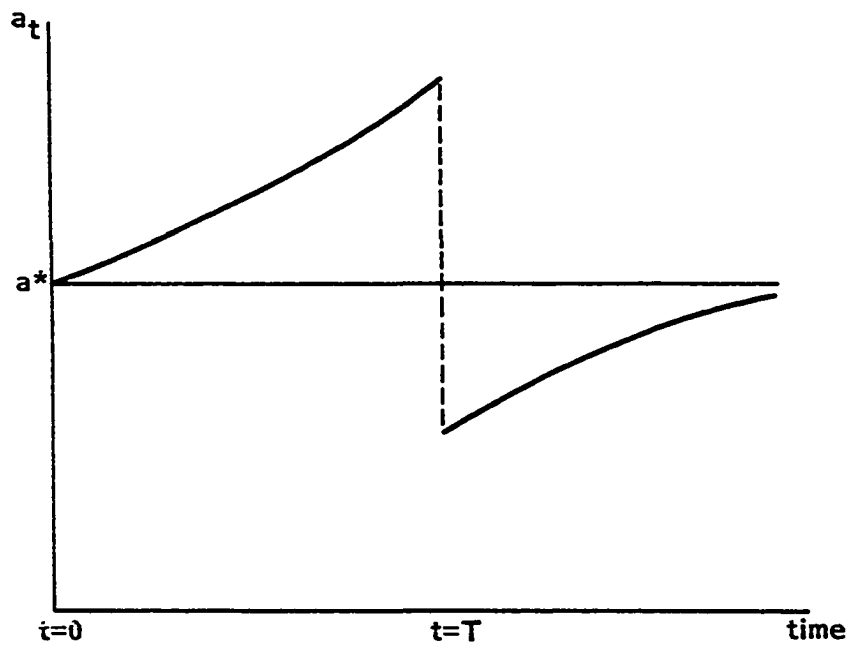


Figure 8b. The time path of the foreign asset level  $a_t$ , under full rational expectations when an open market operation is anticipated

would expect an appreciation in the exchange rate. In order to increase demand for the larger stock of money, the rate of interest must fall, and this occurs due to the expected appreciation in the exchange rate. From then on, appreciation continually occurs until it reaches zero in the steady state.

Using the following assumed parameter values we can simulate the time path of the price level for illustration:  $g = 0.2$ ;  $b_0 = 0.8$ ;  $\tau = 5$ ;  $\lambda = 0.2$ ;  $\theta = 0.5$ ;  $m^* = 100$ ;  $e_0 = 2$ ; and  $e_1 = 2$ . Hence, the calculated values of the remaining parameters are  $r = 1.63$ ;  $s = 0.16$ ;  $x_2 = 0.2$ ;  $\alpha_1 = -1.36$ ;  $\alpha_0 = 0.76$ ;  $\gamma_0 = 96$ ;  $\gamma_1 = -0.02$ ;  $\gamma_2 = 0.81$ ;  $\gamma_3 = -1.47$ ;  $\gamma_4 = 0.15$ ;  $\phi_0 = 6$ ;  $\phi_1 = 0.022$ ;  $\phi_2 = 0.033$ ;  $\phi_3 = 1.58$ ;  $\phi_4 = 0.001$ ;  $D = 1.11$ ;  $\theta_1 = 0.61$ . Let us also assume that the money supply is expected to rise from 100 to 200. The calculated exchange rate levels under partial and full rational expectations are shown in Table 1b below.

The exchange rate overshoots under both partial and full rational expectations. Under full rational expectations, the exchange rate appreciates after  $t = \tau$  and is accompanied by external assets accumulation. Under partial rational expectations when  $a_t$  is treated as exogenous and fixed, the exchange rate does not appreciate after  $t = \tau$ , rather, it stays at that level



Table 1b. Calculated exchange rate levels in response to anticipated open market operations

Time	$S_t$ Full rational expectations	$S_t$ Partial rational expectations
Preannouncement	0.0000	0.0000
t = 0	0.0002	0.0003
t = 1	0.0018	0.0015
t = 2	0.0063	0.0093
t = 3	0.0410	0.0555
t = 4	0.0515	0.3333
t = 5	1.8670	2.0000
t = 6	1.5320	2.0000
t = 7	1.3263	2.0000
t = 8	1.2002	2.0000
t = 9	1.1228	2.0000
t = 10	1.0753	2.0000
t → ∞	$P_t \rightarrow 1.0000$	

permanently. We can conclude then, that what happens to the exchange rate under full rational expectations at  $t = \tau$  (i.e., whether it undershoots or overshoots) depends critically on the structure of the economic system. In particular, if  $\theta$  (fraction of wealth held in the form of real balances) is small, the exchange rate undershoots and if it is large, the exchange rate overshoots its steady state target at  $t = \tau$ .

The effect of a pure monetary expansion produces similar results. Under partial rational expectations a permanent anticipated pure monetary expansion at  $t = \tau$  means that

$$\begin{aligned} dm_0 &= 0; & t + i < \tau \\ &= dm; & t + i \geq \tau \end{aligned}$$

From Equation 18, we have

$$dp_t = \frac{1}{1 + \lambda} \sum_{i=0}^{\infty} \left(\frac{\lambda}{1 + \lambda}\right)^i dm_{t+i}$$

For  $t \leq \tau$ , we have

$$\begin{aligned} dm_{t+i} &\neq 0 && \text{when } t + i \geq \tau \\ &&& \text{or when } i \geq \tau - t \end{aligned}$$

Therefore, the time path of the exchange rate due to a change in expectations at  $t = 0$  is given by

$$dp_t = \left(\frac{1}{1+\lambda}\right) \sum_{i=\tau-t}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^i dm$$

Therefore at  $t = 0$ , the impact effect of the change in expectations is given by

$$dp_0 = \left(\frac{\lambda}{1+\lambda}\right)^\tau dm$$

The time path of the price level (exchange rate) until  $t = \tau$  is given by

$$dp_t = \left(\frac{\lambda}{1+\lambda}\right)^{\tau-t} dm$$

Hence, the longer the time lag between the time of the announcement and the time the change actually occurs, the smaller is the initial depreciation of the exchange rate. For  $t < \tau$ , the exchange rate depreciates exponentially, and the system reaches full neutrality at  $t = \tau$ .

Comparing the impact effect of the unanticipated pure monetary expansion with the anticipated pure monetary expansion effect, we obtain

$$\left. \frac{dS_0}{dME} \right|_{\text{anticipated}} = \left(\frac{\lambda}{1+\lambda}\right)^\tau \left. \frac{dS_0}{dME} \right|_{\text{unanticipated}}$$

and hence

$$\left. \frac{dS_o}{dME} \right|_{\text{anticipated}} < \left. \frac{dS_o}{dME} \right|_{\text{unanticipated}}$$

since  $\left(\frac{\lambda}{1+\lambda}\right)^\tau < 1$

We also realize that the extent of depreciation will be greater for an open market operation than for the pure monetary expansion case, i.e.,

$$\left. \frac{dS_o}{d_{omo}} \right|_{\text{anticipated}} = \left(\frac{1}{1-\theta}\right) \left(\frac{\lambda}{1+\lambda}\right)^\tau > \left. \frac{dS_o}{dME} \right|_{\text{anticipated}} = \left(\frac{\lambda}{1+\lambda}\right)^\tau$$

the reason being that the open market operation is reinforced by the additional effect of the debt retirement; hence, we obtain a stronger decline in asset yields under open market operation.

The analysis on the pure monetary expansion has considered the stock of foreign assets exogenous and constant over time. However, the monetary expansion can affect the stock of A over time through the balance of trade flows. In the full rational expectations model, the analysis is modified to incorporate endogenous wealth changes. A permanent anticipated pure monetary expansion means that

$$dZ_j = 0 \quad \text{for all } j$$

$$\begin{aligned} dm_t &= dm^*, & t &\geq \tau \\ &= 0, & t &< \tau \end{aligned}$$

which implies that

$$\begin{aligned} dV_i &= -dm^*, & i &< \tau \\ &= 0, & i &\geq \tau \end{aligned}$$

The time path of  $a_t$  for  $t \leq \tau$  will be as follows:

(i) for  $t = 0$ ,  $da_0 = 0$

(ii) for  $t < \tau$

$$da_t = \theta^t {}_1 \left[ \sum_{i=0}^{t-1} (\phi_1 r^i + \phi_2 s^i) - \sum_{i=0}^{\tau-t} \phi_2 s^i (r^t - s^t) \right] dm$$

There will be an immediate depreciation of the exchange rate at the time of the announcement of a future increase in the money stock. From then on, the exchange rate will continue to depreciate and the anticipation of depreciation, by lowering real balances, wealth and spending will give rise to a current account surplus, hence,  $da_t > 0$

(iii)  $A_\tau \quad t = \tau$

$$da_\tau = \theta^\tau {}_1 \left[ \sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 s^i) \right] dm$$

(iv) Finally for  $t > \tau$ ,

$$da_t = \theta_1^t \left[ \sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 s^i) dm \right]$$

and as  $t \rightarrow \infty$ ,  $da_t \rightarrow 0$  and therefore after the monetary policy is implemented, the system converges towards its steady state equilibrium. The effect of the pure monetary expansion on the time of the price level is described as follows:

(i)  $t = 0$

$$\begin{aligned} dp_0 &= dm - \sum_{i=0}^{\tau-t} \left[ \gamma_2 + \frac{\phi_2 r^i}{(1+\lambda)r - \lambda} \right] s^i dm \\ &= dm - \sum_0^{\tau-t} \frac{1}{\alpha_0} \left[ b_0 - g + \frac{\lambda g}{(1+\lambda)r - \lambda} \right] s^i dm. \end{aligned}$$

As noted in an earlier discussion, there will be an immediate depreciation of the exchange rate at the time of the announcement of the future increase in the money stock. From then on, the exchange rate will continue to depreciate, and the anticipation of depreciation will lower real balances, real wealth and real expenditures and hence will give rise to current account surplus. The subsequent time paths of the price level under full rational expectations is as follows:

(ii) when  $t < \tau$

$$dp_t = dm - \left[ \frac{r\theta_1^t}{(1+\lambda)r - \lambda} \right] \left[ \sum_{i=0}^{t-1} (\phi_1 r^i + \phi_2 s^i) \right] dm \\ - \sum_{i=0}^{\tau-t} \left[ \gamma_2 + \frac{\phi_2 r \left(\frac{s}{r}\right)^t}{(1+\lambda)r - \lambda} \right] s^i dm$$

(iii) when  $t = \tau$

$$dp_\tau = dm - \left[ \frac{r\theta_1^\tau}{(1+\lambda)r - \lambda} \right] \left[ \sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 s^i) \right] dm$$

Hence no overshooting at  $t = \tau$ .

(iv) Finally when  $t > \tau$

$$dp_t = dm - \left[ \frac{r\theta_1^t}{(1+\lambda)r - \lambda} \right] \left[ \sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 s^i) \right] dm$$

as  $t \rightarrow \infty$ ,  $dp_t \rightarrow dm$  that is the system approaches full neutrality in the long run. The time paths of the price level and the level of external assets are described in Figures 9 and 10. When the change in the money stock is anticipated, the impact effect of the announcement is to generate an immediate depreciation represented by AB (Figure 10), when agents form partial rational expectations. Due to expectations, the exchange rate continues to rise, following the curve BD. The system attains full neutrality at  $t = \tau$  under partial rational

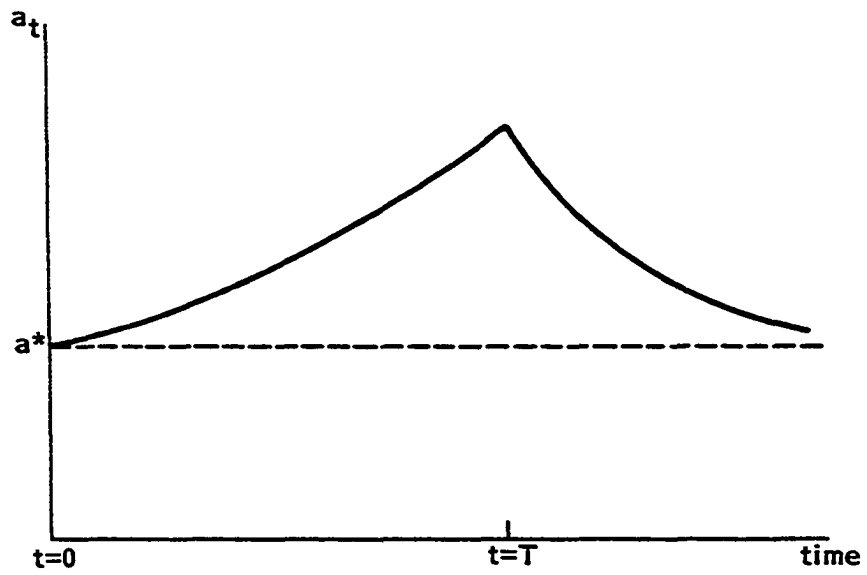


Figure 9. The time path of the external assets level when the change in the money stock is anticipated under full rational expectations



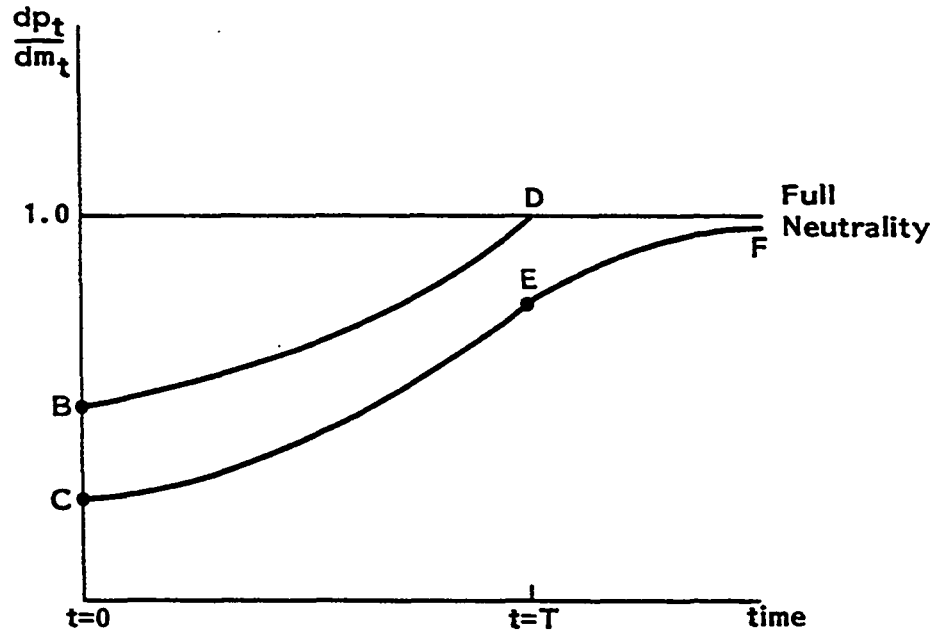


Figure 10. The time paths of the exchange rate when the change in the money stock is anticipated under full rational and partial rational expectations

expectations. Under full rational expectations, the initial jump is represented by AC. Due to expectations, the exchange rate continues to depreciate following the path CE accompanied by current account surplus (refer to Figure 9) and hence foreign asset accumulation. After the money stock is actually increased at  $t = \tau$ , real wealth increases, this now leads to a deficit and decumulation of external assets (Figure 9) until the initial real equilibrium is reattained. Throughout the adjustment process the exchange rate is depreciating. At  $t = \tau$ , the exchange rate cannot be expected to jump, but nominal money does increase, hence from then on agents would expect a decline in the exchange rate depreciation which means a moderation in the rate of increase in the exchange rate. In order to increase the demand for the larger stock of money, the rate of interest must fall, and this occurs due to the expected decline in exchange rate depreciation, after  $t = \tau$ . From then on the exchange rate depreciation continually decreases until it reaches zero in the steady state. If agents, however, treat A as exogenous and constant, then there will not be any offsetting effect of the current account surplus on the exchange rate depreciation, and hence the exchange rate will reach its new steady state at  $t = \tau$ .

It is important to note from the above analysis that purely nominal disturbances have real effects. This is because the anticipated rise in prices and the exchange rate, affect portfolio choices and hence wealth, and the current account. The temporary effects of the anticipated monetary disturbance will have to be reversed before the economy reaches its new steady state. Therefore, the initial current account surplus will then turn to a deficit. Thus, if the government policy can be anticipated at all, changes in the supply of money may generate leads as well as lags in the level of prices and the exchange rate. This phenomenon can alter the timing of shipments of tradeable goods and introduce substantial leads and lags in payments for imports.

Using the same parameter values from the previous example, we can simulate the time path of exchange rate under both partial and full rational expectations.

The simulation results in Table 2 show that after the initial depreciation at  $t = 0$ , the exchange rate continues to depreciate as  $t$  approaches  $\tau$ . Under partial rational expectations, the system attains full neutrality at  $t = \tau$ . Under full rational expectations, the exchange rate rate undershoots its long run target at  $t = \tau$ . There is a moderation in the rate of depreciation of the exchange after the implementation of the monetary policy. The

Table 2. Calculated exchange rate levels in response to anticipated pure monetary expansion

Time	$dS_t/dm_t$ Full rational expectations	$S_t$ Partial rational expectations
Preannouncement	0.00	0.00000
$t = 0$	0.00012	0.00013
$t = 1$	0.00121	0.00077
$t = 2$	0.00250	0.00463
$t = 3$	0.01700	0.02778
$t = 4$	0.02750	0.16667
$t = 5$	0.96720	1.00000
$t = 6$	0.97990	1.00000
$t = 7$	0.98766	1.00000
$t = 8$	0.99243	1.00000
$t = 9$	0.99536	1.00000
$t = 10$	0.99715	1.00000
$t \rightarrow \infty$	$P_t \rightarrow 1.00000$	

exchange rate depreciation continually decreases until it reaches zero in the steady state.

The results in Tables 1 and 2 show that the initial depreciation in the exchange rate under full rational expectations is less than the depreciation in the exchange rate under partial rational expectations. This is because under full rational expectations there is an offsetting wealth effect on the exchange rate depreciation. The results also show that the extent of depreciation is greater for an open market operation than for the pure monetary expansion; the reason being that the open market operation is reinforced by the additional effect of the debt retirement.

For convenience, the results derived in the previous sections are tabulated in Table 3 for comparison. Table 3 shows that at  $t = 0$ , the impact effect of an announcement of monetary expansion is smaller than the impact effect of an unanticipated monetary expansion. This hypothesis will be tested in Chapter 4. In order to carry out this test, anticipated and unanticipated money supplies must be quantified. This involves forecasting the money supply process and separating out expected and unexpected components. The money supply forecast in this study will be carried out by performing time series analysis on the monthly money supplies.

Table 3. Comparison of the effects of anticipated and unanticipated permanent monetary expansion

	$t = 0$	$0 < t < \tau$	$t = \tau$	$t > \tau$
$\frac{dp}{d_{\text{omo}}} \Big _{\text{unantic.}}$	$> 1.0$	approaching 1.0	approaching 1.0	approaching 1.0
$\frac{dp}{d_{\text{ME}}} \Big _{\text{unantic.}}$	$= 1.0$	$= 1.0$	$= 1.0$	$= 1.0$
$\frac{dp}{d_{\text{omo}}} \Big _{\text{antic.}}$	$< 1$	increasing	$< 1$ if $\phi_5 > 0$ $> 1$ if $\phi_5 < 0$	approaching 1.0
$\frac{dp}{d_{\text{ME}}} \Big _{\text{antic.}}$	$< 1$	increasing	$< 1$	approaching 1.0

#### Anticipated transitory monetary disturbance

From Equation 16 it can be shown that the effects of the transitory open market operation and the pure monetary expansion on the spot exchange rate, under partial rational expectations, are a multiple of the effects of the unanticipated transitory monetary disturbances. Hence we have

$$(i) \quad \left. \frac{dS_t}{d_{omo}} \right| TA = \left. \frac{dS_t}{d_{omo}} \right| TU \left( \frac{\lambda}{1+\lambda} \right)^{\tau-t}$$

$$(ii) \quad \left. \frac{dS_t}{dME} \right| TA = \left. \frac{dS_t}{dME} \right| TU \left( \frac{\lambda}{1+\lambda} \right)^{\tau-t}$$

The results suggest that the larger the time lag between the time of the announcement and the time the change actually occurs, the smaller is the initial depreciation in the current exchange rate. The effects of the anticipated increase in the money stock on the exchange rate are greatest in the period in which the change occurs and are proportional to that change in earlier periods with the weights declining geometrically. Afterwards, when there is a corrective monetary policy, the system will return to the original equilibrium.

Endogenous wealth changes and its long run implications were ignored in the above analysis. To take these into consideration we need to examine the analysis under full rational expectations. However, the anticipated transitory monetary expansion could be ignored under full rational expectations, since presumably a transitory effect would be ignored by agents.

General Features of the  
Two Country Model

So far the theory has focused on one country facing a world market. The objective of this section is to formulate an operational theory of determination of one bilateral rate between two countries. The purpose is to obtain a model for the empirical analysis in Chapter 4. We assume there are two countries, A and B. Each country is fully employed, prices in each country are flexible. Each country produces a single identical tradeable good, and the output is given in each country. We also assume the competitive arbitrage assumption about prices that

$$P = SP^* \quad (30)$$

where  $P$ ,  $S$ ,  $P^*$  are as defined before. Equilibrium in the world goods market requires that world real expenditures equal world real income. The implication of the assumption of only a single good is the simultaneous goods market equilibrium and the total real income and total real expenditures balance, hence

$$E + E^* = Y + Y^* \quad (31)$$

where

$$E = E(Y, W) \quad (32)$$

$$E^* = E^*(Y^*, W^*) \quad (33)$$



$E$  ( $E^*$ ) is the domestic (foreign) real expenditures. These depend on real income and real wealth. It is assumed that there are only four types of assets: domestic currency, foreign currency, domestic index bonds each yielding one unit of domestic output per unit of time, and foreign index bonds each yielding one unit of foreign output per unit of time. Let us assume that domestic bonds are perfect substitutes for the foreign bonds in the portfolio of individual wealth holders. Let us also assume that they are issued by the domestic government. At each point in time, wealth holders in each country allocate their stock of wealth between bonds and domestic money, hence

$$W = \frac{M}{P} + \frac{A}{r^*} \quad (34)$$

$$W^* = \frac{M^*}{P^*} + \frac{A^*}{r^*} \quad (35)$$

where  $A$  ( $A^*$ ) is the domestic (foreign) ownership of the real asset;  $r^*$  is the endogenously determined world real interest rate; and  $W$  ( $W^*$ ),  $M$  ( $M^*$ ) and  $P$  ( $P^*$ ) are as defined before.

Due to perfect capital mobility, we have

$$\begin{aligned} r_N^A &= r^* + \dot{P}/P \\ r_N^B &= r^* + \dot{P}^*/P^* \end{aligned}$$

hence

$$r_N^A - r_N^B = x$$

where  $r_N^A$  ( $r_N^B$ ) is the country A (B) nominal interest rate. It is assumed that in each country the fraction of real wealth which residents wish to hold in real balances depends on the alternative cost of holding money and real income, so that

$$\frac{M^d}{P} = \theta(r^* + \pi, Y)W \quad (36)$$

$$\frac{M^{*d}}{P^*} = \theta^*(r^* + \pi^*, Y^*)W^* \quad (37)$$

where  $\pi = \dot{P}/P$  and  $\pi^* = \dot{P}^*/P^*$ . It is also assumed that the fraction of real wealth which residents in each country wish to hold in bonds depends on the interest rate and on real income. The wealth constraint requires that the sum of their real demands for all assets must be equal to their stock of wealth. Also, the sum of the partial effects on the two asset demands of a change in either the interest rate or real income must be zero. Thus, only one of the two asset demand functions is independent in each country. That is, since the asset demand functions are dependent on wealth, and because of the wealth constraint, and also given perfect substitutability between bonds,

only one of the asset demand functions is independent. Therefore, equilibrium in the assets markets will be described by specifying equilibrium in the money markets only. Substituting Equation 34 into 36 and 35 into 37 we obtain the following two equations:

$$\begin{aligned} \frac{M}{P} &= \left[ \frac{\theta(r^* + \pi, Y)}{1 - \theta(r^* + \pi, Y)} \right] \frac{A}{r^*} \\ &= \frac{A}{r^*} \cdot L(r^* + \pi, Y) \quad L_1 < 0, L_2 > 0 \end{aligned} \quad (38)$$

and

$$\begin{aligned} \frac{M^*}{P^*} &= \left[ \frac{\theta^*(r^* + \pi^*, Y^*)}{1 - \theta^*(r^* + \pi^*, Y^*)} \right] \frac{A^*}{r^*} \\ &= \frac{A^*}{r^*} \cdot L^*(r^* + \pi^*, Y^*) \quad L^*_1 < 0, L^*_2 > 0 \end{aligned} \quad (39)$$

The complete model of the system then is

$$\frac{M}{P} = \frac{A}{r^*} L(r^* + \pi, Y) \quad (40)$$

$$\frac{M^*}{P^*} = \frac{A^*}{r^*} \cdot L^*(r^* + \pi^*, Y^*) \quad (41)$$

$$E + E^* = Y + Y^* \quad (42)$$

At any point in time,  $M$ ,  $M^*$ ,  $Y$ ,  $Y^*$ ,  $A$ ,  $A^*$  and the expectational variable are predetermined, and the three equations determine the short run variables  $P$ ,  $P^*$  and  $r^*$ . Having solved for  $P$  and  $P^*$  we can then determine the exchange rate, using the purchasing power parity condition.

### The Exchange Rate Equation

Using Equations 40 and 41 we can obtain

$$\frac{M}{M^*} = \frac{PA \cdot L(r^* + \pi, Y)}{P^*A^* \cdot L^*(r^* + \pi^*, Y^*)} \quad (43)$$

Assuming the demand for money in both countries is of the conventional type, Equation 43 can be specified as

$$\frac{M}{M^*} = \frac{PAY^\gamma e^{-\lambda(r^* + \pi)}}{P^*A^*\gamma^*\gamma^* e^{-\lambda^*(r^* + \pi^*)}} \quad (44)$$

Assuming that the domestic and foreign parameters are the same, that is assuming symmetry,  $\gamma = \gamma^*$  and  $\lambda = \lambda^*$  we shall have

$$\frac{M}{M^*} = S \left( \frac{A}{A^*} \right) \left( \frac{Y}{Y^*} \right)^\gamma e^{-\lambda x} \quad (45)$$

where  $x$  is the expected rate of change of the exchange rate. Taking logs on both sides yields

$$S = (m - m^*) + (a^* - a) + \gamma(y^* - y) + \lambda x \quad (46)$$

Where  $x$  is a proxy for the forward premium, that is  $x_t = E_t (S_{t+1} - S_t)$  where  $S_{t+1}$  ( $S_t$ ) is the logarithm of the forward (spot) exchange rate and  $E_t$  denotes the expectations operator. From Equation 46 the bilateral exchange rate depends on the foreign and domestic money supplies, stock of assets owned by both the domestic and foreign residents, outputs in both countries and the expected rate of change of the exchange rate. For a given stock of real assets and outputs in each country, the rate at which the exchange rate depreciates is equal to the difference in the rates of change in nominal money supplies. Also, when the domestic real income grows faster than the foreign real income, the domestic currency will appreciate. Growth in real income stimulates the demand for money, and with given nominal stock of money, this requires a fall in the price level and hence, exchange rate appreciation. Similarly, an increase in the accumulation of bonds in the domestic country, for example through current account surplus raises real wealth and consequently leads to exchange rate appreciation. And finally, an expected exchange rate depreciation reduces the demand for real money, the price level therefore rises and the exchange rate depreciates to maintain purchasing

power parity. The anticipation of depreciation therefore induces an actual depreciation.

Following the same procedure as in Section 1, we can derive the rational expectations solution for the exchange rate, and obtain

$$S_t = \frac{1}{1+\lambda} \sum_{j=0}^{\infty} E_t[(m-m^*)_{t+j} + (a^*-a)_{t+j} + \gamma(y^*-y)_{t+j}] \left(\frac{\lambda}{1+\lambda}\right)^j$$

The current exchange rate then depends not only on the domestic current and expected future monetary and real variables, but also on the foreign current and future behavior of these variables. Again we can examine the effects of unanticipated and anticipated monetary policies on the exchange rate. The end results will be qualitatively the same as found in Section 1. But the adjustment mechanism in this case will be different. In the small country case, the interest rate remains unchanged and the expected rate of exchange depreciation serves as the equilibrating mechanism. In this general equilibrium model, the world interest rate does not remain unchanged. The interest rate responds to both domestic and foreign monetary policies, and hence affects their saving activities, and hence the price levels and the exchange rate.

The results arrived at in this chapter will be tested in the following chapter. Data from the U.S., Germany and Netherlands will be used.

## CHAPTER 4. EMPIRICAL ESTIMATION

The objective of the first part of this chapter is to examine the empirical validity of the simple asset market model of exchange rate determination in Equation 47. As has been pointed out, the model is based on an assumption concerning the interrelationships between domestic and foreign prices, through the condition of purchasing power parity, and by manipulating money market equilibrium conditions. Studies by Bilson (1978), Clements and Frenkel (1980) and Branson, Halttunen and Masson (1977) follow this approach.

Bilson (1978) tested his model using monthly data for the Federal Republic of Germany and the United Kingdom over a period from April 1970 to May 1977. In order to incorporate some form of distributed lag mechanism to take account of the slow adjustment of the actual exchange rate to the equilibrium exchange rate level, he assumed that the actual exchange rate adjusts toward the equilibrium rate according to the equation  $\ln(S) - \ln(S_{-1}) = \gamma[\ln(\underline{S}) - \ln(S_{-1})]$  where  $\gamma$  denotes the partial adjustment coefficient and  $\underline{S}$  denotes the equilibrium exchange rate, that is defined as

$$S = \frac{M}{M^*} \left[ \frac{Y}{Y^*} \right]^{-\eta} \left[ \frac{K^*}{K} \right] e^{\varepsilon(i-i^*)}$$



He also allows for a trend in the shift factor, as specified below.

$$\ln(K/K^*) = K_0 + K_1 t$$

where  $K_0$  is a constant and  $K_1$  is the rate of growth in the relative money demand. Hence, the exchange rate equation becomes

$$\begin{aligned} \ln(\underline{S}) = & \beta_0 + \beta_1 \ln(M) + \beta_2 \ln(M^*) + \beta_3 (i - i^*) + \beta_4 \ln(Y) \\ & + \beta_5 \ln(Y^*) + \beta_6 t + \beta_7 \ln(S_{-1}) + V \end{aligned} \quad (47)$$

where

$$\begin{aligned} \beta_0 &= \gamma K_0; \quad \beta_1 = \gamma; \quad \beta_2 = -\gamma; \quad \beta_3 = \gamma \epsilon; \quad \beta_4 = -\gamma \eta; \\ \beta_5 &= \gamma \eta; \quad \beta_6 = \gamma K_1; \quad \beta_7 = 1 - \gamma. \end{aligned}$$

He estimated Equation 47 in two ways; first he estimated the equation as it is without any restrictions on the coefficients and secondly imposed stochastic prior restrictions on the coefficients. For convenience, the results he obtained are as stated in Tables 4 and 5. Observation of the unrestricted model leads to the condition that the data do not support the monetary model of exchange rate determination based upon the fact that none of the coefficients of the money supply

Table 4. OLS estimates for Ln(S) Dm/ unrestricted

	$\beta_0$	$\text{Ln}(M_t)$	$\text{Ln}(M^*_t)$	$(i-i^*)$	$\text{Ln}(Y_t)$	$\text{Ln}(Y^*_t)$	t	$\text{Ln}(S-1)$
Ln(S)	-1.15	0.46	0.04	0.34	-0.03	-0.18	-0.01	0.71
	(0.765)	(1.446)	(0.457)	(2.670)	(0.218)	(1.099)	(2.321)	(9.27)
	SE = 0.0258		$R^2 = 0.9832$		$\rho = 0.2087$		DW = 1.954	

Table 5. OLS estimates for Ln(S) DM/ restricted-mixed sample and prior information

	$\beta_0$	$\text{Ln}(M_t)$	$\text{Ln}(M^*_t)$	$(i-i^*)$	$\text{Ln}(Y_t)$	$\text{Ln}(Y^*_t)$	t	$\text{Ln}(S-1)$
Ln(S)	-0.25	0.19	-0.18	0.26	-0.17	0.19	-0.001	0.81
	(0.901)	(5.072)	(5.172)	(2.741)	(3.135)	(3.362)	(3.055)	(23.20)
	SE = 0.0276		$R^2 = 0.9807$		$\rho = 0.2496$		DW = 1.9707	

or real income variables are statistically significant. The problem, according to Bilson, may be that many of the variables on the right hand side of the equation are highly collinear, so that the sample information cannot distinguish their influence on the exchange rate. He then imposed the prior information consistent with the monetary theory on the model and obtained the results shown in Table 5. The results as shown are consistent with the predictions of the monetary theory. The consistency of the equation with the predictions of the monetary approach is best seen by deriving the associated expression for the equilibrium exchange rate. The equilibrium rate is derived by setting  $S_t$  equal to  $S_{t-1}$ . Doing this he obtained the results shown below in Table 6.

Clements and Frenkel (1980) have applied the monetary approach to the exchange rate determination to the analysis of the monthly dollar/pound exchange rate over the period February 1921 to May 1925, during which time exchange rates were flexible. Special attention was given to the relationship between the exchange rate and the relative price of traded and nontraded goods. Since there are no available data for the prices of traded and nontraded goods, they proxied the prices of traded goods by the wholesale price indices and the prices of nontraded goods by wages. In general, the empirical results they

Table 6. OLS estimates for Ln(S) DM/ restricted-mixed sample and prior information

	$\beta_0$	$\text{Ln}(M_t)$	$\text{Ln}(M^*_t)$	$(i-i^*)$	$\text{Ln}(Y_t)$	$\text{Ln}(Y^*_t)$	$t$
$\text{Ln}(\underline{S})$	-1.328	1.003	-0.985	1.385	-0.901	1.018	-0.005
		(6.259)	(6.258)	(2.792)	(3.341)	(3.623)	(3.247)

Table 7. OLS estimates for Ln(S) \$/

	$\beta_0$	$\text{Ln} \frac{P_T/P_N}{P^*_T/P^*_N}$	$\text{Ln}(M)$	$\text{Ln}(M^*)$	$\text{Ln}(Y/Y^*)$	$(i-i^*)$
$\text{Ln}(S)$	-4.297	0.415	1.050	-0.044	0.188	0.363
	(1.396)	(0.099)	(0.1821)	(0.143)	(0.066)	(0.350)
	$R^2 = 0.96$	$\text{SE} = 0.015$	$\text{DW} = 1.55$		$\rho = 0.88$	

obtained were reasonably satisfactory, in relation to the predictions of the theoretical model. Results are shown in Table 7.

The figures in parentheses are the standard errors of the estimated coefficients. The coefficient of relative prices turned out with the predicted sign and highly significant. The elasticity of the exchange rate with respect to the domestic money supply is consistent with the homogeneity postulate, that a given change in the supply of money results in an equiproportionate change in the exchange rate. However, the elasticity with respect to the foreign money supply does not differ significantly from zero, meaning that the set of data rejects the restriction of equality between domestic and foreign elasticities. They attributed the inadequate fitting of the data to the model to the following reasons: (a) differences in definitions of the U.S. and U.K. money supplies used in the monthly series. (b) U.K. monetary series have varied much less than that of the U.S.

The estimated elasticity of the exchange rate with respect to the income ratio turned out to be positive and significant. The positive sign is in contrast with the prediction of the monetary model. Finally, the coefficient on the interest rate differential had the positive sign predicted by the model, but the

parameter estimate did not differ significantly from zero.

Branson, Halttunen and Masson (1977) have applied the asset market model empirically to the dollar/deutsche mark exchange rate. They examined a bilateral model for the \$/DM rate as a function of measures of the U.S. and West German stocks of money and net foreign assets. They also extended the theory to include government reaction functions for both monetary policy and exchange market intervention. They argue that government actions, by altering the stocks of assets held by the private sector, will necessarily, affect the exchange rate. They argue further that accounting for policy actions merely through the addition of exogenous variables to the model is not enough, rather the model must incorporate additional equations explaining the systematic part of the authorities behavior. They included policy reaction functions for international reserves of Germany and for domestic component of the German Central bank money stock. Their estimation results are presented in Table 8, where  $S$  = spot exchange rate;  $M1$  = money stock in domestic currencies;  $FP$  = private foreign assets stocks in dollars. Figures in parentheses are the  $t$  - statistics.

The first equation shows the ordinary least squares estimates, and the second shows the Cochrane-Orcutt

Table 8. OLS estimates for S (\$/DM)

	Mlg	Mlu	FPg	FPu	RHO	R <sup>2</sup>	DW
S	-9.43 (-6.0)	-0.103 (-6.1)	0.143 (10.5)	0.249 (1.1)	-0.289 (1.8)	0.826	0.41
S	-8.8 (-0.2)	-0.057 (-1.6)	0.089 (2.8)	0.456 (1.3)	-0.293 (-1.6)	0.864 (13.7)	1.33

estimates, correcting for serial correlation in the error terms. All the coefficients in the first equation have the predicted signs and t-statistics for the MIs are fairly high; but the residuals are highly correlated. The Cochrane-Orcutt equation shows a RHO value of 0.866 with only the U.S. money stock Mlu remaining significant at the 5 per cent level. The coefficients still have the expected signs but the MI coefficients are cut by half. The coefficient for FPu is fairly stable but that for FPg increases by a factor of nearly two. The residuals in the equation with Cochrane-Orcutt transformation still exhibits some autocorrelation.

In Table 9 below, we have consistent estimates of reaction functions estimated jointly with the exchange

Table 9. Consistent estimates for reaction functions and exchange rates (25LS)

		<u>Central bank money and M1 (in DM)</u>							
		$\beta_0$	$\overline{MB}_g^a$	e(\$/DM)	RHO	R <sup>2</sup>	DW		
(1a)	MBg	-0.736 (4.6)	1.074 (72.9)	80.140 (1.3)		0.994	0.87		
(1b)	MBg	-0.785 (-1.8)	1.080 (26.8)	-67.053 (-1.5)	0.731 (6.3)	0.997	1.508		
(1c)	Mlg	0.959 (0.2)	1.372 (3.5)	-67.944 (-0.6)	0.917 (13.6)	0.989	1.719		
		<u>Foreign exchange reserves (in \$)</u>							
		$\beta_0$	FGg-1	e(\$/DM)	RHO	R <sup>2</sup>	DW		
(2)	FGg	11944 (2.8)	0.631 (4.7)	188.673 (2.8)	0.338 (2.1)	0.780	1.629		
		<u>Exchange rate</u>							
		$\beta_0$	Mlg	Mlu	FPg	Fpu	RHO	R <sup>2</sup>	DW
(3)	S	151.58 (1.5)	-0.02 (-0.4)	0.0145 (0.3)	0.338 (0.7)	-0.247 (-0.8)	0.814 (8.4)	0.771	1.118

<sup>a</sup>Where MB and  $\overline{MB}$  are central bank money and target central bank money, respectively.



rate equation. Two stage least squares were used on the three-equation system.

In the estimated Equations 1a and 1b of Table 9 for German base money, the realized value of the base is very closely related to the target, but it is weakly related to the exchange rate change with a positive coefficient. When the equation is estimated with the Cochrane-Orcutt transformation the coefficient for the exchange rate change takes on a negative sign, which is also insignificant. Equation (1c) has M1 as the dependent variable, and there is no effect running from the exchange rate to money. These results support the findings of other studies that the Bundesbank has generally been able to pursue its control of the money supply very closely without interference from the exchange rate vis-à-vis the dollar.

In the estimated reaction function for interventions (Equation 2 in Table 9), the foreign exchange reserve stock is positively and significantly related to the change in the exchange rate, indicating an attempt to smooth fluctuations.

Many other researchers have applied the monetary approach to exchange rate determination to other data sets from several other countries. Results are mixed in nature. But a general conclusion that can be drawn

from the existing empirical literature is that the monetary model explains a very high percentage of the variation in the exchange rate, however these results do not support the consideration of the monetary model as a complete description of the exchange rate. This may be due to several factors. For example, the implications of the policy actions of governments as they influence the exchange rate is not examined in the typical asset approach model. Incorporation of additional equations explaining the systematic part of the authorities' behavior would help improve upon the estimation results. Secondly, the exchange rate, together with other macro-economic variables, are determined in a general equilibrium framework by the interaction of flow and stock conditions. Hence the asset market equation may be too simple to capture all the influences on the exchange rate. The theoretical analysis under a set of assumptions may be correct, even though its empirical form is inadequate to fit the facts.

#### The Model

The general features of a bilateral exchange rate model relate to assumptions concerning equilibrium in the money markets in the two countries, and the condition of purchasing power parity. In this section the bilateral

exchange rate model developed in Chapter 3 will be tested using data from the U.S., Germany and Netherlands. The money demand functions for the domestic country (U.S.) and the foreign country (Germany) are specified as:

$$\frac{M^d}{P} = AY^\gamma e^{-\lambda(r^* + \pi)} \quad (48)$$

$$\frac{M^{*d}}{P^*} = A^*Y^*\gamma e^{-\lambda(r^* + \pi^*)} \quad (49)$$

From Equations 48 and 49, we obtain

$$\frac{M^d}{M^{*d}} = \frac{PAY^\gamma e^{-\lambda(r^* + \pi)}}{P^*A^*Y^*\gamma e^{-\lambda(r^* + \pi^*)}} \quad (50)$$

Equation 50 is modified to include a time trend in the relative money demands. Hence we have

$$\frac{M^d}{M^{*d}} = \frac{PAY^\gamma e^{-\lambda(r^* + \pi)}}{P^*A^*Y^*\gamma e^{-\lambda(r^* + \pi^*)}} e^{\delta T} \quad (51)$$

This means that the time series is randomly fluctuating around an average level that changes in a linear or straight-line fashion over time. Hence,  $\delta$  provides an estimate of the trend in relative demand for money, since

$$\delta = \frac{d \ln(M^d/M^{*d})}{dT} = \frac{1}{M^d/M^{*d}} \cdot \frac{d(M^d/M^{*d})}{dT}$$

In equilibrium, the demand for real money equals the real money supply. These two real money market equilibrium conditions, together with the purchasing power parity yield the estimating exchange rate equations, in log linear form.

$$S = \beta_0 + \beta_1(m-m^*) + \beta_2(a^*-a) + \beta_3(y^*-y) + \beta_4x - \beta_5T + U$$

From the theoretical analysis,  $x = E_t[(S_{t+1}-S_t)/S_t]$ , hence the forward premium on exchange rates can be used, since it is predominantly influenced by speculative factors, and hence is a better empirical proxy for the type of interest rate stressed by monetary theory. In actual fact, the interest rate is not an independent variable; the interest rate, and the exchange rate are simultaneously determined in a more general equilibrium model. Therefore, there is a problem of simultaneous bias in the estimation equation, when interest rates or when forward premium are used.

The equation will be estimated in two ways, first it will be estimated without imposing any of the restrictions of equality between the domestic and foreign

parameters. And secondly, the restrictions will be imposed and the equation estimated.

### Empirical Results

The time period undertaken by this study is from January 1972 to December 1980. One hundred and eight observations of monthly data are used for the empirical work within the above time period. Exchange rates began to float in 1972 and hence, the reason for choosing the above time period. Monthly data on all variables were obtained from the International Financial Statistics, published by the International Monetary Fund.

The exchange rate equation is estimated using ordinary least square regression procedure. The results are presented in Table 10 below. This is the unrestricted model. The values for the multiple correlation coefficient ( $R^2$ ), the standard error (SE), the first order autocorrelation ( $\hat{\rho}$ ) and the Durbin-Watson Statistic (DW) are listed for the estimated equation.

The elasticity of the exchange rate with respect to the domestic money supply has the expected sign but is not significant. The elasticity of the exchange rate with respect to the domestic real income has the predicted sign and is also significant, however, with respect to the foreign real income, the exchange rate

Table 10. Unrestricted OLS estimates for S(DM/\$)

Variable	Parameter/ Estimate	T-ratio	Prob >  T
Intercept	-0.713	-0.502	0.616
M	0.149	0.685	0.494
M*	0.262	0.792	0.429
Y	-0.435	-1.611	0.110
Y*	0.025	0.107	0.914
A	0.167	2.490	0.014
A*	0.211	4.594	0.001
FP	0.438	0.511	0.611
T	-0.013	-6.192	0.001
$R^2 = 0.9167$	S.E = 0.003	$\hat{\rho} = 0.748$	DW = 0.48

elasticity even though it has the predicted sign, is insignificant. The parameter estimate on the real bonds held by German residents also turned out with a sign which is in contrast to the prediction of the asset market model, but however, very insignificant. The estimate on the real bonds held by the U.S. residents has the predicted sign and is highly significant. The coefficient on the forward premium has the predicted

sign but slightly insignificant. This may be due to the fact that the forward premium is not a good proxy for the interest rate differential. Aliber (1973) points out that the interest rate parity theorem will not hold if the assets are issued in different countries and there is some exchange control. And Hodrick (1976) claims that controls were adopted in Germany in February 1973 as a result of massive capital inflows into Germany at the end of the Bretton Woods fixed-parity system. The effect of these controls was to drive a wedge between the real interest rate in Germany and the real interest rate in the U.S., and the rest of the world. The coefficient on the time trend variable has the predicted sign and is highly significant, showing that there is actually a significant trend in the relative demands for money and hence in the exchange rate. In general, the results obtained are not reasonably satisfactory, compared to the predictions of the model. The model, however, explains over 90% of the variation in the exchange rate, with a standard error of 0.003.

The problem of serial correlation is a frequent one when using time series data. The stochastic disturbance terms in part reflect variables not included explicitly in the model, and these may change slowly over time. It can therefore be expected that the stochastic disturbance

term at one observation may be related to the stochastic disturbance terms at nearby observations. Serial correlation results in least squares estimates that are not efficient and also results in the failure of the usual statistical tests of significance.

The Durbin-Watson statistic from the equation estimated indicate the presence of first-order serial correlation in the residuals. Hence, the ordinary least squares estimation procedure is inefficient. Such first-order serial correlation takes the form of a first-order autoregressive scheme. The following relationship then are assumed to hold for the disturbance terms:

$$U_t = \rho U_{t-1} + V_t \quad \text{all } t, \quad |\rho| < 1$$

where  $\rho$  is the first order auto correlation coefficient of the U series; and  $V_t$  is a residual stochastic disturbance term, which is assumed to satisfy the assumptions of the basic regression model, including absence of serial correlation:

$$\begin{aligned} E(V_t) &= 0 \\ E(V_t, V_{t+s}) &= \gamma^2 v && \text{for } s = 0 \\ &= 0 && \text{for } s \neq 0 \end{aligned}$$



To correct for the serial correlation in the residuals, the original model has to be transformed. One of the most accurate procedures is the Durbin's two-stage procedure. First the matrices  $Y$ ,  $X$  and  $U$  are transformed in the following manner:

$$TY = TX + TU$$

The  $T$  matrix is approximated by

$$T = \begin{bmatrix} -\rho & 1 & 0 & \dots & \dots & \dots & 0 \\ 0 & -\rho & 1 & \dots & \dots & \dots & 0 \\ \cdot & & & & & & \\ \cdot & & & & & & \\ \cdot & & & & & & \\ 0 & 0 & 0 & \dots & -\rho & 1 & 0 \\ 0 & 0 & 0 & \dots & 0 & -\rho & 1 \end{bmatrix}$$

In matrix form, the original model is written as

$$Y_z = X_t \beta + U_t \quad (52)$$

Lagging one period and multiplying by  $\rho$ , yields

$$\rho Y_{t-1} = \rho X_{t-1} \beta + \rho U_{t-1} \quad (53)$$

subtracting 53 from 52 we obtain

$$Y_t - \rho Y_{t-1} = (X_t - \rho X_{t-1})\beta + U_t - U_{t-1} \quad (54)$$

and hence

$$Y_t = \rho Y_{t-1} + (X_t - \rho X_{t-1})\beta + V_t \quad (55)$$

The estimated value of  $\rho$  is obtained by fitting Equation 55 and the calculated value for  $\rho$  is then substituted into the T matrix. The AUTOREG PROCEDURE, available in SAS can be used to estimate the parameters of a linear model whose error term is assumed to be an autoregressive process.

The exchange rate equation without any parameter restrictions, and corrected for the first order autocorrelation is shown in Table 11 below. After correcting for the first order autocorrelation among the residuals, the results are significantly improved. In particular, the elasticities of the exchange rate with respect to domestic and foreign money supplies have the predicted signs. The elasticities with respect to the domestic and the foreign real incomes do not have the signs predicted by the model; however, they are not statistically significant. The elasticities with respect to the domestic and foreign real bond holdings turned out with the signs predicted by the model, but are not statistically different from zero. The parameter estimate for the forward

Table 11. Unrestricted estimates for S(DM/\$) corrected for first order autocorrelation

Variable	Parameter estimate	T-ratio	Prob >  T
Intercept	0.636	0.363	0.717
M	0.226	1.800	0.074
M*	-0.087	-0.436	0.663
Y	0.036	0.177	0.859
Y*	-0.088	-0.333	0.740
A	-0.037	-0.577	0.565
A*	0.078	1.158	0.249
FP	-0.767	-0.988	0.325
T	-0.007	-3.009	0.003
$R^2 = 0.638$		S.E = 0.001	

premium also turned out with a sign contrary to what is predicted by the model, but also not significant. The coefficient on the T variable has the predicted sign and highly significant.

Multicollinearity is a problem in a study of this nature, because the variables are very likely to be correlated across countries. The insignificance of the

coefficients may be due in part to the strong correlation among explanatory variables. In Table 12 below, estimates of the coefficients are presented when equality restrictions between domestic and foreign parameters are imposed and when correlation for the first order autocorrelation is also made.

Table 12. Estimates for S(DM/\$) restrictions imposed, corrected for first order autocorrelation

Variable	Parameter estimate	T ratio	Prob >  T
Intercept	1.564	11.562	0.0001
(M - M*)	0.600	2.845	0.0054
(Y* - Y)	-0.421	-1.964	0.0523
(A* - A)	0.062	1.858	0.0661
FP	0.580	0.706	0.4820
T	-0.1006	-13.251	0.0001
$R^2 = 0.898$		S.E = 0.003	

Imposing the restriction of equality between domestic and foreign parameters improves the results a great deal. As shown in Table 12, the money supply elasticities are close to the prediction of the model, and are highly significant at the 5 per cent significance level; and it also has the sign predicted by the model. The elasticities of the exchange rate with respect to domestic and foreign values of bond holdings are also consistent with the prediction of the theory, and highly significant at the 10 per cent significance level. The coefficient on the forward premium turned out with the sign predicted by the model, but it is not significantly different from zero. The parameter estimate for the time trend also turned out with the predicted sign and is highly significant at the 5 per cent significance level. However, the exchange rate elasticities with respect to domestic and foreign real incomes have signs opposite to those predicted by the model, and are significantly different from zero at the 10 per cent level. As has been explained already, this might be due to the poor proxies for the real income variable. The model explains nearly 90 per cent of the variation in the exchange rate and has a standard error of 0.003. The overall improvement in the estimation may be attributed to the reduction of the problem of multicollinearity, by imposing the parameter restrictions.

The model is also tested using monthly data for the Netherlands and the United States over a period from January 1972 to December 1980. The results are presented in Table 13 below.

Table 13. Estimates for S(G/\$) restrictions imposed, corrected for first order autocorrelation

Variable	Parameter estimate	T ratio	Prob >  T
Intercept	1.912	14.319	0.0001
(MN - M*)	0.840	4.793	0.0001
(Y* - YN)	0.222	1.475	0.1434
(A* - AN)	0.059	3.794	0.0003
FPN	0.759	1.234	0.2199
T	-0.006	-16.231	0.0001
$R^2 = 0.911$		S.E = 0.002	

where MN = Netherlands money supply  
 YN = Netherlands real income  
 AN = Netherlands holdings of external bonds  
 FPN = Forward premium on exchange rate (G/\$)

All the parameter estimates turned out with the signs predicted by the asset market approach model. The money supply coefficient turned out to be 0.84, which is consistent with the homogeneity postulate predicted by the model, and also highly significant at the 5 per cent level. The elasticity of the exchange rate with respect to real income is 0.222, but not significantly different from zero at the 5 per cent level. The coefficient on real bond holdings turned out to be very significant at the 5 per cent level and a parameter coefficient of 0.059. The coefficient on the forward premium is also not significant at the 5 per cent level. The model explains over 91 per cent of the variation in the exchange rate and has a standard error of 0.002.

#### Anticipated and Unanticipated Money Supplies and the Exchange Rate Level

The objective of this section is to test whether unanticipated and anticipated money changes have explanatory value for the exchange rate. In order to carry out this test, anticipated and unanticipated money supplies must be quantified. This involves forecasting the money supply process and separating out expected and unexpected components. A similar approach has been adopted by Barro (1977 and 1978). In his studies, he analyzes

the effects of anticipated and unanticipated money growth on output (GNP) and the price level (GNP deflator) for recent U.S. experience. In his formulation, the money-growth rate is related to a measure of federal government expenditure relative to normal (which captures an aspect of the revenue motive for money creation), a lagged measure of the unemployment rate (which reflects countercyclical response of money growth), and two annual lagged values of money growth (which pick up persistence effects not captured by the other explanatory variables). In this study, the money supply forecast is carried out by performing time series analysis on the monthly money supplies. To be consistent with the theory in Chapter 3, we have to work with the logarithms of the money supplies. Hence,  $\ln(M_t)$  forms the working series for the time series analysis. Basically, what this involves is the use of the  $\ln(M_t)$  series to predict the logarithm of the money supply  $\ln(\hat{M}_t)$  and the residuals  $R = \ln(M_t) - \ln(\hat{M}_t)$ . These are the anticipated and unanticipated components of the money supply. These are used to determine their effects on the exchange rate. In practice, regression models are frequently applied with good results to forecasting time series with dependent or autocorrelated observations. However, since the Box-Jenkins methodology uses the dependency in the observations more effectively than do



regressions, the Box-Jenkins methodology is likely to produce more accurate forecasts than the forecasts produced by the regression approach (Bowerman and O'Connell, 1979). Moreover, the Box-Jenkins methodology offers a more systematic approach to building, analyzing, and forecasting with time series models. The Box-Jenkins methodology consists of a three step iterative procedure. The first step is identification. In this step a tentative model is identified by analysis of the historical data. The second step is estimation. In this step the unknown parameters of the tentative model are estimated. The third step is diagnostic checking. At this stage, diagnostic checks are performed to test the adequacy of the model, and to suggest potential improvement. The three steps described are illustrated below using the German series.

#### Identification

This section deals with obtaining information about a time series, leading to identification of a Box-Jenkins model. Examination of the sample autocorrelation function of the  $\ln(M_t)$  series in Figure 11 indicates stationarity, since the sample autocorrelation function for the time series dies down rapidly. The sample partial autocorrelation function in Figure 12 cuts off at lag 2, indicating



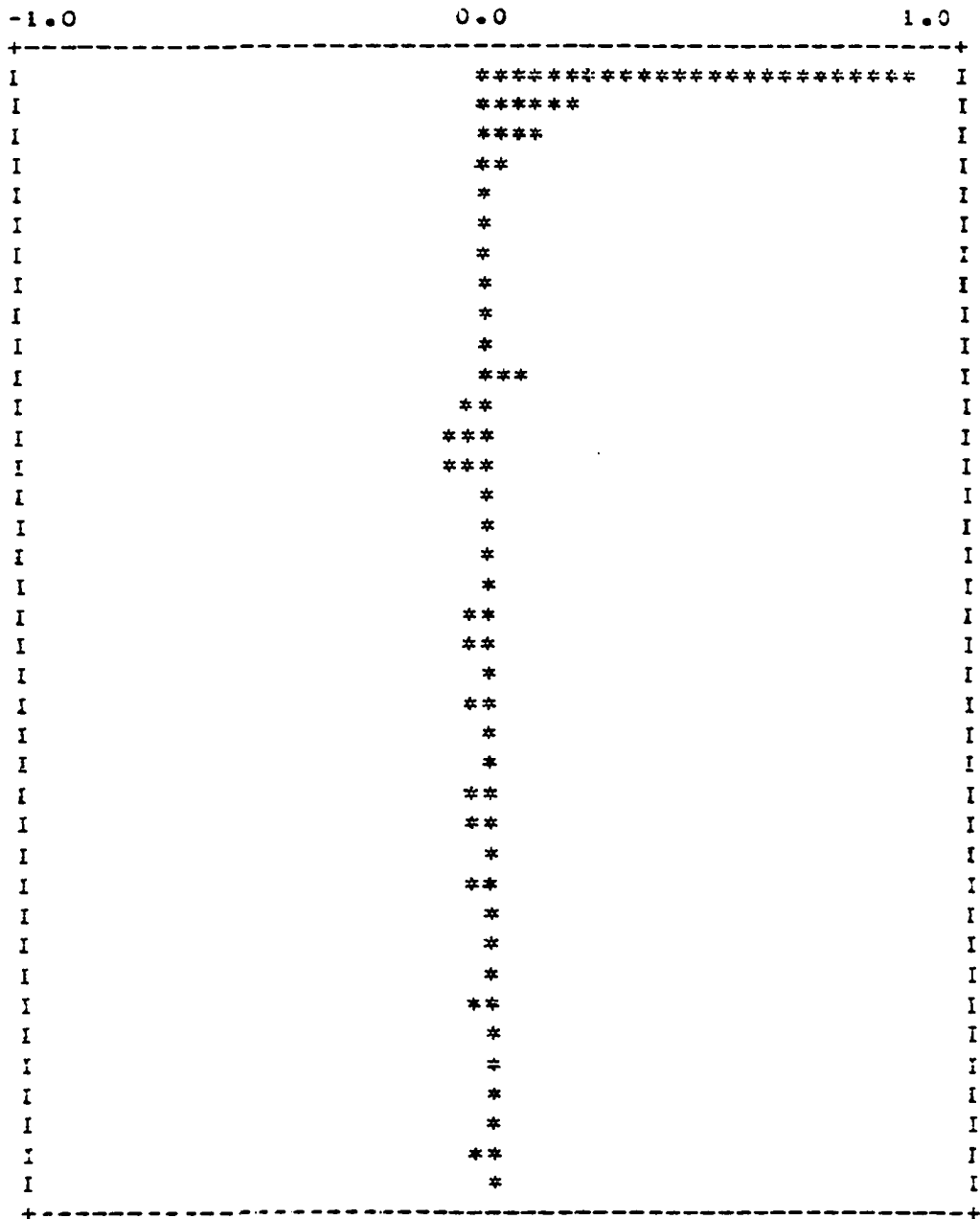


Figure 12. Sample partial autocorrelation function for time series  $\text{Ln}(M_t)$

that possibly, an autoregressive process of order 2 (AR(2)), might adequately describe the  $\text{Ln}(M_t)$  series. For some time series applications, the appropriate Box-Jenkins model can be identified correctly after only a simple examination of the sample autocorrelation and sample partial autocorrelation functions of the time series. However, more often than not, especially with seasoned time series, one will generally have two or more candidate models that require further comparison. To start with, we will compare two models, AR(1) and AR(2).

Model 1: Autoregressive process of order 1

$$Z_t = \delta + \phi_1 Z_{t-1} + \varepsilon_t \quad (56)$$

where  $Z_t = \text{Ln}(M_t)$  and  $\varepsilon_t$  is the error term.

Model 2: Autoregressive process of order 2

$$Z_t = \delta + \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \varepsilon_t \quad (57)$$

#### Estimation and Diagnostic Checking

In order to compare and choose among the candidate models, it is necessary to estimate the parameters of each of them and to examine their properties. The diagnostic checking procedures adopted here are based on the analysis of residuals. These procedures provide more opportunity for the data themselves to suggest modifications.

Autocorrelation check

Outputs pertaining to the residuals from the fitted model are useful for diagnostic checking. An effective way to measure the overall adequacy of the tentative model is to examine a quantity that determines whether the first  $K$  autocorrelations of the residuals considered together, indicate adequacy of the model. This is the Box-Pierce chi-square statistic, and is computed using the formula

$$a = (n-d) \sum_{i=1}^K r_i^2(\epsilon)$$

where  $n$  = number of observations in the original time series

$d$  = the degree of differencing that was used to transform the original time series into a stationary time series.

$r_i^2(\epsilon)$  = the square of  $r_i(\epsilon)$ , the sample autocorrelation of the residuals at lag  $i$ , that is, the sample autocorrelation of residuals separated by a lag of  $i$  time units.

The modelling process is supposed to account for the relationships between the observations. If it does account for these relationships, the residuals should be unrelated, and hence the autocorrelations of the residuals should be small. Thus, a large value indicates that the model is inadequate.

Significant autocorrelations in residuals from fitted models indicate that the error terms are related, and hence the model is not adequate enough. Such problems are usually corrected by adding moving average terms to account for the correlations in the error terms.

The adequacy of a Box-Jenkins model can also be judged by considering the quantity.

$$\text{Standard Error } S = \sqrt{\frac{\text{SSE}}{n-p}}$$

where  $n$  = number of observations in the original time series.

$p$  = the number of parameters that must be estimated in the model.

$S$  measures the overall fit of the model. The smaller  $S$  is, the better the overall fit is considered to be.

#### Cumulative periodogram check

In fitting time series, it is important to adequately take into account the periodic characteristics of the series. A periodogram is specifically designed for the detection of periodic patterns. The periodogram of a time series  $a_t$ ,  $t = 1, 2, \dots, n$  is defined as

$$I(f_i) = \frac{2}{n} \left[ \left( \sum_{t=1}^n a_t \cos 2\pi f_i t \right)^2 + \left( \sum_{t=1}^n a_t \sin 2\pi f_i t \right)^2 \right]$$

where  $f_i = i/n$  is the  $i$ th harmonic of the fundamental frequency  $1/n$ . We shall refer to  $c(f_j)$  as the normalized cumulative periodogram

$$c(f_j) = \frac{1}{nS^2} \sum_{i=1}^j I(f_i)$$

where  $S^2$  is an estimate of  $\sigma_a^2$ . For a white noise series, the plot of  $c(f_j)$  against  $f_j$  would be scattered about a straight line. On the other hand, model inadequacies would produce non-random  $a$ 's whose cumulative periodogram could show systematic deviations from this line. Marked departures from linearity in the cumulative periodogram plot of the residuals indicate periodicities inadequately taken account of.

Summary results for models 1 and 2 are presented in Table 14. Table 14 presents the estimates of the parameters in each model, also  $t$ -statistics for each of the estimates are given in the parentheses below the estimates. The table also gives the Box-Pierce chi-square statistic (with 20 degrees of freedom) for each of the fixed models and also indicates lags at which the residuals possess significant autocorrelations. Both models have high Box-Pierce  $X^2$  values, and hence indicate model inadequacy. From Figures 13 and 14, we realize that both autocorrelations in residuals from fitted models indicate that the error

Table 14. Comparison of models 1 and 2

	Model	
	1	2
No. of regular differences	0	0
No. of seasonal differences	0	0
No. of parameters	1	2
$\phi_1$	0.9964 (40.27)	0.6654 (7.11)
$\phi_2$		0.3329 (3.57)
$\delta$	0.018	0.0083
Box-Pierce $X^2$ (20 D.F.)	19.412	15.384
Significant autocorrelations in residuals	Lag 1	Lag 2
Standard Error	0.0036	0.0032
$R^2$	0.936	0.944

terms are related. The autocorrelation function of the residuals from AR(1) model has a significant t-value at lag 1, as seen in Figure 13. This suggests that a regular moving average of order 1 (MA(1)) term need be added to the AR(1) model. The autocorrelation function



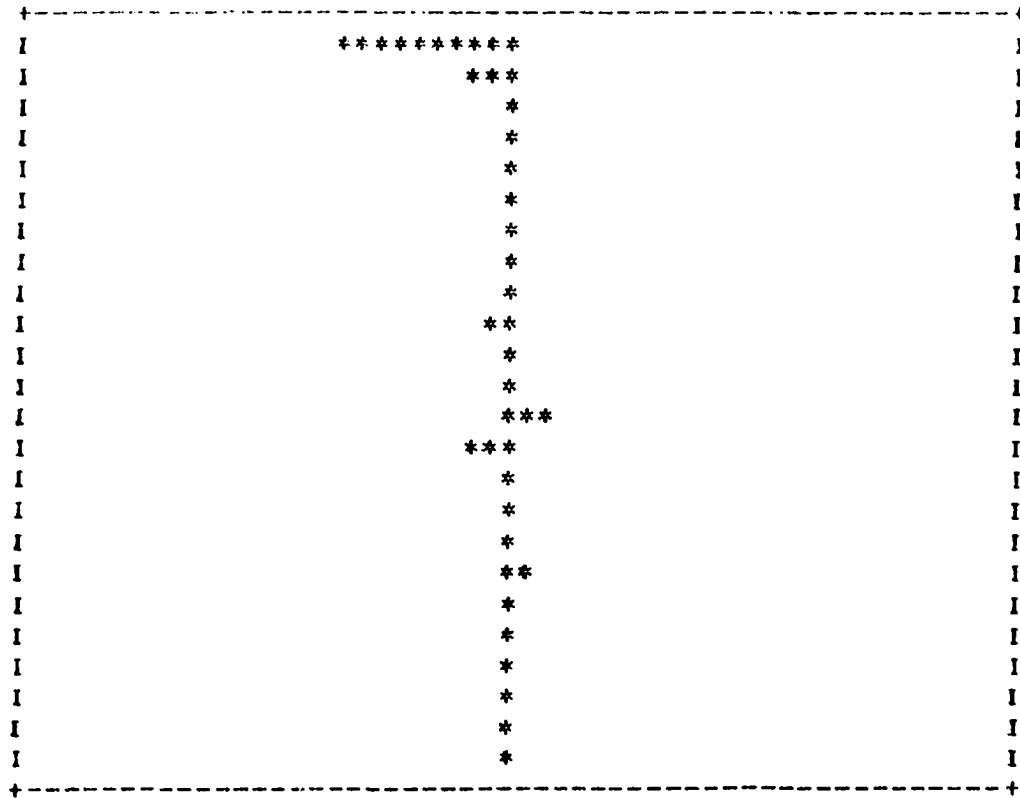


Figure 13. Autocorrelation function of the residuals for model AR(1)

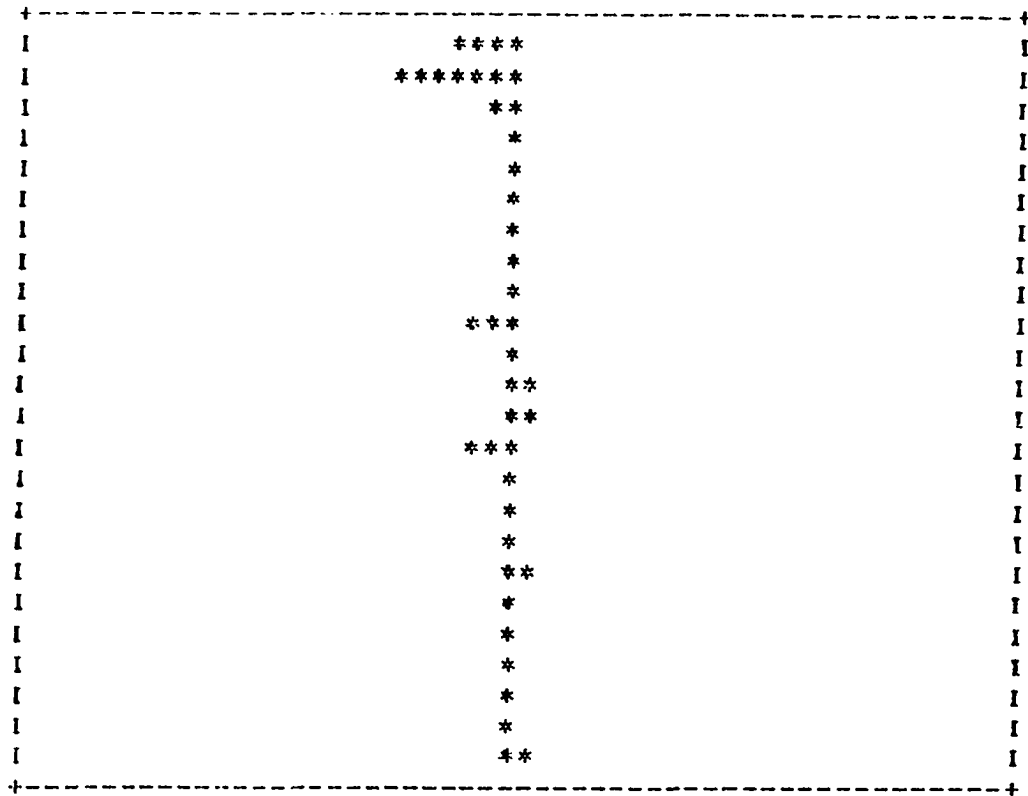


Figure 14. Autocorrelation function of the residuals for model AR(2)

of the residuals from the AR(2) model also has a significant t-value at lag 2 as shown in Figure 14, which also suggests that a regular moving average term needs to be added to the AR(2) model. Figures 15 and 16 show the cumulative normalized periodograms of the residuals of the AR(1) and AR(2) models, respectively. These two models show significantly departures from linearity, and hence periodic characteristics have not been taken account of in these models. In view of these inadequacies in the AR(1) and AR(2) models, a more accurate model has to be used. The significant t-values of the autocorrelations of the residuals from both models suggests that a regular moving average term has to be added to the models to account for the correlations in the error terms. An addition of a regular moving average term MA(1) to AR(1) to form an ARMA(1, 1) model produced the best results. The standard error estimate was reduced to 0.003 and the Box-Pierce  $X^2$  statistic was also reduced to 9.55. It explains 94.8 per cent of the variation in the  $\text{Ln}(M_t)$  series. From Figure 17 we observe that the residuals from the ARMA(1, 1) model has no significant autocorrelation, hence the model adequately takes account of all the correlations in the error terms. From Figure 18, the normalized periodogram of the residuals does not show any significant departure from linearity, and hence the periodic characteristic

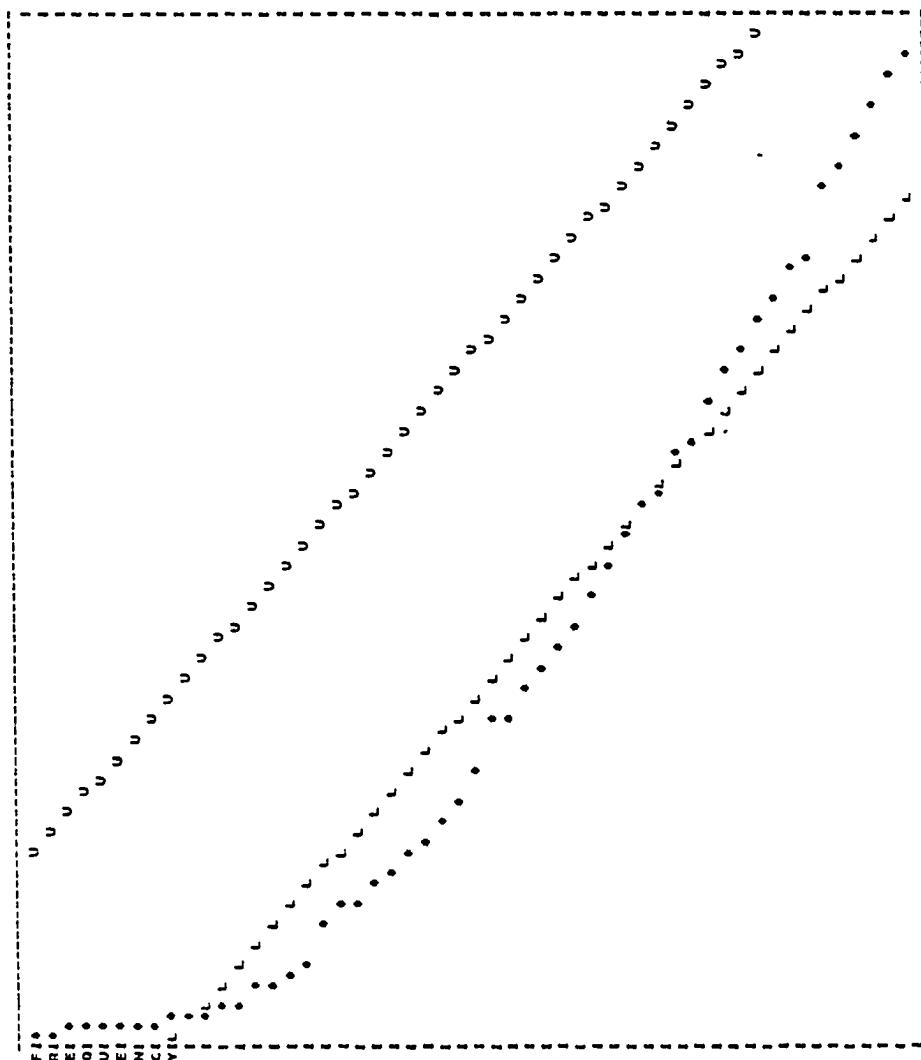


Figure 15. Cumulative normalized periodogram for model AR(1)



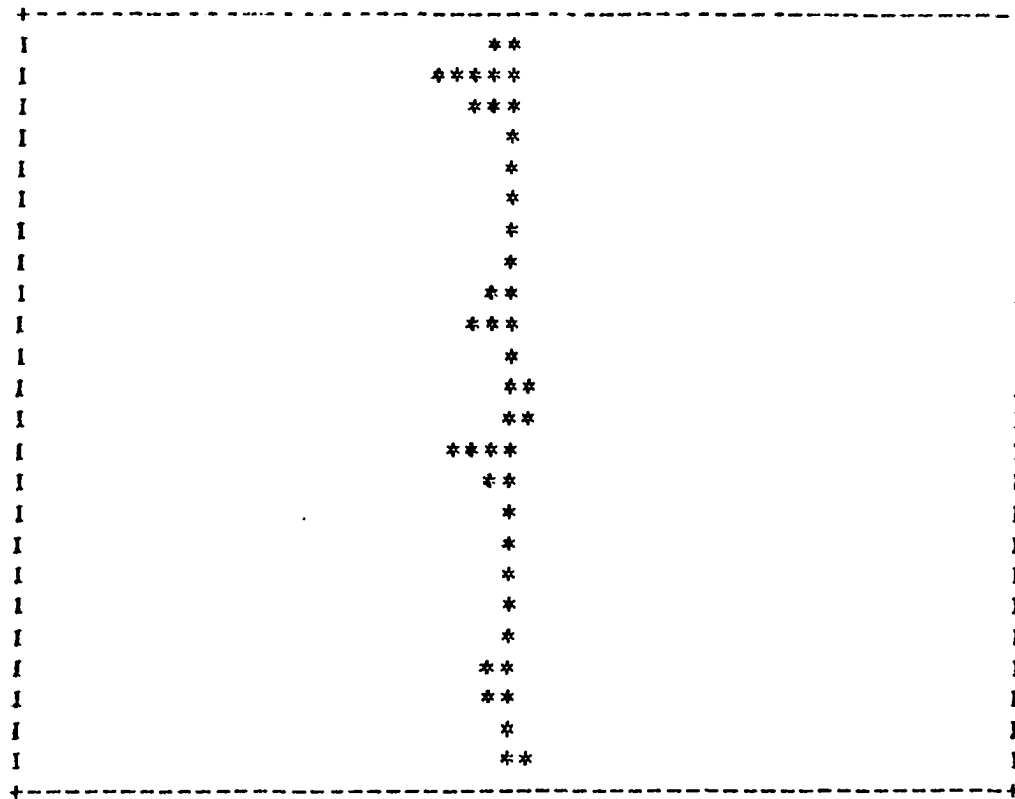


Figure 17. Autocorrelation function of the residuals for the model ARMA(1, 1)

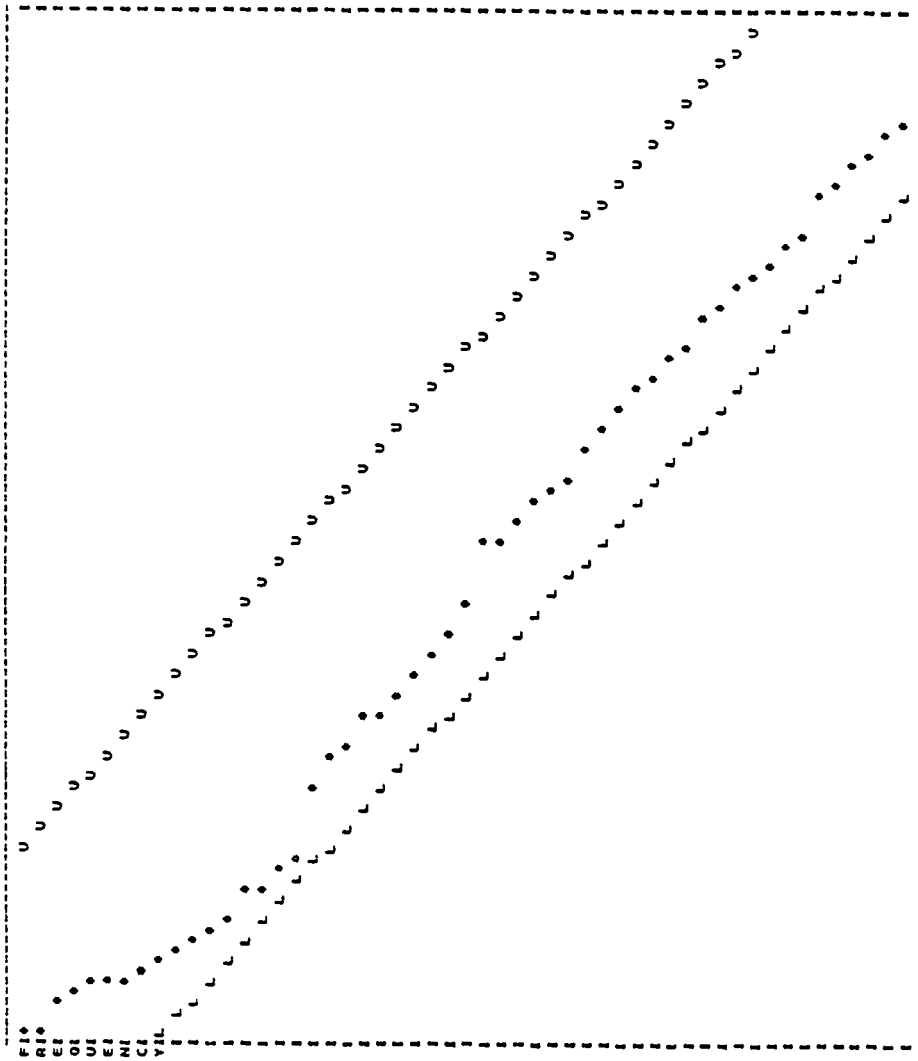


Figure 18. Cumulative normalized periodogram for model ARMA(1, 1)

has been taken care of. The estimated model parameters yield the following prediction equation.

$$Z_t = 0.0029 + 0.9994Z_{t-1} - 0.5246\varepsilon_{t-1} + \varepsilon_t \quad (58)$$

(93.28)                      (6.06)

Box-Pierce  $X^2(20 \text{ D.F.}) = 9.55$        $R^2 = 0.948$        $DW = 1.91$

The estimated values from Equation 58,  $\hat{Z}_t$ , and the residuals  $R_t = Z_t - \hat{Z}_t$  are used to measure the anticipated and unanticipated components of money supply, respectively. In the theoretical analysis of Chapter 3, we established that the qualitative effects of unanticipated and anticipated monetary changes are the same, they both lead to spot rate depreciation, however the quantitative effects are not the same. The extent of depreciation due to unanticipated money changes is much greater than depreciation due to anticipated monetary disturbance. The purpose of this section is to test this hypothesis. To achieve this, we have to regress  $\ln(S_t)$  on  $Z_t$ , the anticipated component;  $R_t$  the unanticipated component; and the other explanatory variables discussed in Chapter 3. Table 15 below shows the empirical results for the U.S.-German data.

The results clearly support the hypotheses derived in Chapter 3 that the extent of depreciation due to unanticipated monetary changes is much greater than that



Table 15. OLS estimates for anticipated and unanticipated monetary changes for S(DM/\$)

Variable	Parameter estimate	T-ratio	Prob >  T
Intercept	3.661	2.804	0.006
$\hat{Z}$	0.163	0.800	0.426
R	0.269	1.981	0.050
M*	-0.624	-2.561	0.012
Y* - Y	-0.328	-1.434	0.155
A* - A	0.174	2.038	0.044
FP	-0.136	-0.147	0.884
T	-0.002	-1.484	0.141
$R^2 = 0.889$		S.E = 0.004	

where  $\hat{Z}$  = anticipated component of German money supply

R = unanticipated component of German money supply

All other variables are as defined before.

due to anticipated changes. The t-statistics show that the coefficient on  $\hat{Z}$  is not statistically significant, however the coefficient on R is statistically significant at the 5 per cent level. The overall fit is quite

satisfactory, an  $R^2$  value of 0.889 and a standard error of 0.004 were obtained.

The same procedure was also used for the Netherlands-U.S. data. The prediction equation for the money supply is

$$\hat{Z}N_t = 0.0007 + ZN_{t-1} + 0.4893\varepsilon_{t-1} + \varepsilon_t$$

(89.4467) (5.2187)

where  $ZN$  = anticipated component of the Netherlands money supply

$RN = ZN - \hat{Z}N$  = Unanticipated component of the Netherlands money supply.

The effects of  $\hat{Z}N$  and  $RN$  on the exchange rate (G/\$) are presented in Table 16 below. The Netherlands-U.S. data also support the hypothesis arrived at in Chapter three coefficients on  $\hat{Z}N$  and  $RN$  are highly significant at the 5 per cent level. All other explanatory variables have the predicted signs. The overall fit is also good. An  $R^2$  of 0.911 and a standard error of 0.002 were obtained.

#### Exchange Rate Volatility

The purpose of this section is to test the hypothesis of positive correlation between observed exchange rates

Table 16. OLS estimates for anticipated and unanticipated monetary changes for  $S(G/\$)$ 

Variable	Parameter estimate	T-ratio	Prob >  T
Intercept	0.483	0.453	0.651
ZN	0.404	5.501	0.001
RN	0.445	5.513	0.001
M*	-0.139	-0.783	0.435
Y* - YN	0.048	1.142	0.256
A* - AN	0.061	3.577	0.005
FPN	0.973	1.489	0.139
T	-0.007	-5.599	0.001
$R^2 = 0.911$		S.E = 0.002	

and the imprecision in the expectations of the time paths of the exogenous policy variables, as discussed in Chapter 3. The imprecision in the expectations of the time paths of the exogenous policy variables is approximated by the money supply deviation from the expected money supply. The test is conducted using data from Netherlands and Germany and the results are presented in Table 17 below.

The correlation coefficients are shown in Table 17 and the figures in parentheses show the significance

Table 17. Correlation between exchange rates and money supply deviation from expected money supply

---

	R	RN
S	0.031 (0.754)	
SN		0.0101 (0.918)

where S = (DM/\$) exchange rate

R = German money supply deviation from the expected money supply.

SN = Netherlands-U.S. (G/\$) exchange rate

RN = Netherlands money supply deviations from the expected money supply.

---

probability of a correlation coefficient. This is the probability that a value of the correlation coefficient as large or larger in absolute value than the one calculated would have arisen by chance, were the two random variables truly uncorrelated. These results indicate that the data decisively reject the hypothesis of correlation between the two variables, even though the correlation coefficients are all positive as predicted.

## CHAPTER 5. SUMMARY AND CONCLUSIONS

This study set out to examine the effects of unanticipated and anticipated monetary policies on the economy, and to test the derived hypotheses. In the model developed in Chapter 3, it is clear that the value of a floating exchange rate is mainly determined by conditions of asset market equilibrium, given existing stocks of money, domestic assets, and foreign assets. In our model, the excess of income over expenditures is equal to the rate at which the home country acquires claims on the rest of the world. If a country is running a surplus on the trade balance, so that net foreign assets are increasing, this tends to cause the exchange rate to appreciate, and a deficit in the trade balance with net foreign assets falling, causes the exchange rate to depreciate. This is the key to dynamic adjustment of the exchange rate as we move from short run to long run.

Under rational expectations, the exchange rate equation establishes that the current spot exchange rate is determined by all of the current and expected future values of the exogenous variables. Hence, it is necessary to distinguish between anticipated and unanticipated changes in the exogenous variables. Hence, in response to an anticipated change in the money supply, the equilibrium exchange rate

adjusts in advance of the expected increase in the money supply because the demand for money is affected by the expected rate of inflation which affects the domestic interest rate. Since the absolute price level will be higher at  $t = \tau$  (time money is expected to increase), the inflation rate between  $t = 0$  and  $t = \tau$  must be higher than was previously expected. This increase in the expected inflation rate reduces the demand for money at every date up to  $t = \tau$ , thereby requiring an increase in the absolute price level and hence the exchange rate at every date up to  $t = \tau$ . The anticipation of depreciation lowers real balances, wealth and expenditures, and hence gives rise to a current account surplus. The process continues up until  $t = \tau$ , the time the money supply increases. From then on, the exchange rate depreciation continues but the increased real balances, wealth and spending now lead to a deficit and decumulation of foreign assets until the initial real equilibrium is reattained. When the monetary expansion is by open market operation, then whether the exchange rate overshoots or undershoots its steady state target at  $t = \tau$  depends on the economic structure. In particular, if  $\theta$  (the fraction of real wealth held in the form of real balances) is small, the exchange rate undershoots and if  $\theta$  is large, the exchange rate overshoots its steady state target.

An unanticipated change in the stock of money that is expected to persist is neutral. If it is an open market operation, the exchange rate depreciates more than in proportion to the increase in the money stock. As the system converges towards the new equilibrium, foreign assets are accumulated until the current account is back in balance.

These hypotheses derived in Chapter 3 are tested in Chapter 4, using U.S.-German and U.S.-Netherlands data sets. The results obtained from the two data sets indicate that the data decisively support the hypotheses that the changes in the exchange rates observed immediately after a policy shift are larger, the greater the extent to which the policy shift catches economic participants by surprise. The data sets also explain in each case about 90 per cent of the variability in the exchange rate, and each set also supports the conclusion that monetary expansion leads to currency depreciation in the short run.

The formulation of the present model reflects the recent development by Dornbusch and Fischer (1980). In relation to other recent studies, it is interesting to note that these results reinforce the results of other studies. In particular, Wilson (1979) analyzes the effect of temporary anticipated monetary expansion and finds that the larger the increase in the money stock,

the larger the instantaneous impact on the exchange rate and the further into the future it is expected that the increase will occur, the less the current impact. Similar results are also implied by Brock (1975), who analyzes anticipated monetary policy in the context of a perfect foresight monetary model.

Some care must be taken in the interpretation of the dependence of the exchange rate on the various explanatory variables. This model assumes that all the explanatory variables are exogenous within the framework. And when this does not hold, the application of an ordinary least squares estimation procedure results in biased estimates. This is because the exchange rate together with some of these variables are simultaneously determined in a more general equilibrium framework.

Many of the exchange rate determination models have been used to explain the effects of monetary expansion, both in the short run and in the long run and virtually none of the models adds significantly to our insights about the effects of fiscal policy on exchange rates. An improved understanding of exchange rate behavior requires better models not only of fiscal policies but also of the process of wealth accumulation, role of policy mixes, and also of exchange rate expectations. This study hopes



to have provided some answers to some of these vital questions.

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## APPENDIX 1. DATA

## Time Period Under Study

The time period undertaken by this study is from January 1972 to December 1980. One hundred and eight observations of monthly data are used for the empirical work within the above time period. Exchange rates began to float in 1972 and hence the reason for choosing the above time period.

## Variables and Definitions

- S(DM/\$) These series are period averages of deutsche mark/dollar exchange rates quoted as units of German Mark per U.S. dollar.
- SN(G/\$) Period averages of guilder-dollar exchange rates quoted as units of guilder per U.S. dollar.
- M German money supply (M1) in billions of marks.
- M\* U.S. money supply (M1) in billions of dollars.
- MN Netherlands money supply (M1) in billions of guilders.
- Y German industrial production index (1975 = 100). These indices are used as proxies for the levels of real national income, since they are widely used as leading indicators of gross national product.
- Y\* U.S. industrial production index (1975 = 100).

YN	Netherlands industrial production index (1975 = 100).
A	German holdings of real U.S. bonds.
A*	U.S. holdings of real U.S. bonds.
AN	Netherlands holdings of real U.S. bonds.
FP	Forward premium on the deutsche mark-dollar exchange rate.
FPN	Forward premium on the guilder-dollar exchange rate.

Derived variables

Z	Predicted German money supply $\text{Ln}(\hat{M}_t)$
ZN	Predicted Dutch money supply $\text{Ln}(\hat{MN}_t)$
R	Unpredicted German money supply $\text{Ln}(M_t) - \text{Ln}(\hat{M}_t)$
RN	Unpredicted Dutch money supply $\text{Ln}(MN_t) - \text{LN}(\hat{MN}_t)$

Data Sources

Monthly data on all variables are obtained from the International Financial Statistics, published by the International Monetary Fund.

APPENDIX 2. DERIVATION OF THE FORMULA FOR  
 $x_t$  (EXPECTED RATE OF DEPRECIATION)

$$x_t = E_t \left[ \frac{S_{t+1} - S_t}{S_t} \right]$$

$$= \frac{\Delta S}{S}$$

Using the Taylor series

$$\ln \left[ 1 + \frac{\Delta S}{S} \right] = \frac{\Delta S}{S} - \frac{1}{2} \left( \frac{\Delta S}{S} \right)^2 + \frac{1}{3} \left( \frac{\Delta S}{S} \right)^3 - \dots$$

Hence

$$\frac{\Delta S}{S} = \ln \left[ 1 + \frac{\Delta S}{S} \right] + \frac{1}{2} \left( \frac{\Delta S}{S} \right)^2 - \frac{1}{3} \left( \frac{\Delta S}{S} \right)^3 + \dots$$

$$\approx \ln \left[ 1 + \frac{\Delta S}{S} \right]$$

if we assume higher powers to  
be relatively small.

$$= \ln \left[ \frac{S + \Delta S}{S} \right]$$

$$= \ln \left[ \frac{S_{t+1}}{S_t} \right]$$

Hence,

$$x_t = \ln(S_{t+1}) - \ln(S_t)$$



APPENDIX 3. DERIVATION OF PARAMETER VALUES  
FOR EQUATIONS DERIVED UNDER FULL RATIONAL EXPECTATIONS

The two equations obtained under full rational expectations are:

$$p_{t+1} - (1 + \lambda)p_t - a_t = -V_t - (e_1 + m^*)$$

$$a_{t+1} - ba_t - gp_t = -gV_t - Z_{t+1} + (e_0 - gm^*)$$

These two equations can be collapsed into one equation to form

$$\begin{aligned} \lambda p_{t+2} - (1+\lambda+\lambda b_0)p_{t+1} + [b_0(1+\lambda)-g]p_t \\ = (b_0-g)V_t - V_{t+1} - Z_{t+1} + (b_0-1)(e_1+m^*) + (e_0-gm^*) \end{aligned} \quad (59)$$

or

$$\begin{aligned} \lambda a_{t+2} - (1+\lambda+\lambda b_0)a_{t+1} + [b_0(1+\lambda)-g]a_t \\ = -\lambda gV_{t+1} + g\lambda V_t - \lambda Z_{t+2} + (1+\lambda)Z_{t+1} - (e_0 + ge_1) \end{aligned} \quad (60)$$

In the steady state,  $p_{t+2} = p_{t+1} = p_t = p_t^*$ ;  $m_{t+1} = m_t = m^*$ ;  
 $V_{t+1} = V_t = 0$  and  $Z_t = 0$ .

Hence,

$$p^* = m^* + \frac{(b_o - 1)e_1 + e_o}{b_o - 1 - g}$$

$$= \gamma_o$$

Consider the equation

$$\lambda - r(1 + \lambda + \lambda b_o) + r^2[(1 + \lambda)b_o - g] = 0$$

and define

$$\alpha_2 = \lambda$$

$$\alpha_1 = -(1 + \lambda + \lambda b_o)$$

$$\alpha_o = (1 + \lambda)b_o - g$$

Hence,

$$\alpha_o + \alpha_1 + \alpha_2 = b_o - g - 1$$

The roots of the quadratic equation are

$$r = \frac{-\alpha_1 + \sqrt{(\alpha_1)^2 - 4\alpha_o\alpha_2}}{2\alpha_o}$$

$$s = \frac{-\alpha_1 - \sqrt{(\alpha_1)^2 - 4\alpha_o\alpha_2}}{2\alpha_o}$$

Conjecture at a solution of the form

$$P_t = \gamma_0 + \gamma_1 \sum_0^{\infty} V_{t+i} r^i + \gamma_2 \sum_0^{\infty} V_{t+i} S^i + \gamma_3 \sum_0^{\infty} Z_{t+1+i} r^i + \gamma_4 \sum_0^{\infty} Z_{t+1+i} S^i \\ + \gamma_5 \theta^t 1$$

$$\text{Define } N_1 = \sum_0^{\infty} V_{t+2+i} r^i \quad N_2 = \sum_0^{\infty} V_{t+2+i} S^i$$

$$M_1 = \sum_0^{\infty} Z_{t+3+i} r^i \quad M_2 = \sum_0^{\infty} Z_{t+3+i} S^i$$

Hence

$$N_1 = \sum_0^{\infty} V_{t+2+i} r^i = V_{t+2} + V_{t+3} r + V_{t+4} r^2 + \dots$$

and

$$\sum_0^{\infty} V_{t+1+i} r^i = V_{t+1} + V_{t+2} r + V_{t+3} r^2 + \dots \\ = V_{t+1} + r N_1$$

and

$$\sum_0^{\infty} V_{t+i} r^i = V_t + V_{t+1} r + V_{t+2} r^2 + V_{t+3} r^3 + \dots \\ = V_t + V_{t+1} r + r^2 N_1$$

We can do similarly for  $N_2$ ,  $M_1$  and  $M_2$ . Hence the second order differential Equation 59 becomes

$$\begin{aligned}
& \alpha_2[\gamma_0 + \gamma_1 N_1 + \gamma_2 N_2 + \gamma_3 M_1 + \gamma_4 M_2] \\
& + \alpha_1[\gamma_0 + \gamma_1 r N_1 + \gamma_2 S N_2 + \gamma_3 r M_1 + \gamma_4 S M_2 + (\gamma_1 + \gamma_2) V_{t+1} \\
& \qquad \qquad \qquad + (\gamma_3 + \gamma_4) Z_{t+2}] \\
& + \alpha_0[\gamma_0 + \gamma_1 r^2 N_1 + \gamma_2 S^2 N_2 + \gamma_3 r^2 M_1 + \gamma_4 S^2 M_2 + (\gamma_1 + \gamma_2) V_t \\
& \qquad \qquad \qquad + (\gamma_3 + \gamma_4) Z_{t+1} + (\gamma_1 r + \gamma_2 S) V_{t+1} + (\gamma_3 r + \gamma_4 S) Z_{t+2}] \\
& = (b_0 - g) V_t - Z_{t+1} - V_{t+1} - (b_0 - 1)(e_1 + M^*) + e_0 - gm^*
\end{aligned}$$

Rearranging terms:

$$\begin{aligned}
& \gamma_0(\alpha_0 + \alpha_1 + \alpha_2) + \gamma_1 N_1(\alpha_2 + \alpha_1 r + \alpha_0 r^2) \\
& \quad + \gamma_2 N_2(\alpha_2 + \alpha_1 S + \alpha_0 S^2) \\
& \quad + \gamma_3 M_1(\alpha_2 + \alpha_1 r + \alpha_0 r^2) \\
& \quad + \gamma_4 M_2(\alpha_2 + \alpha_1 S + \alpha_0 S^2) \\
& \quad + \alpha_0(\gamma_1 + \gamma_2) V_t + \alpha_0(\gamma_3 + \gamma_4) Z_{t+1} \\
& \quad + \alpha_0(\gamma_1 r + \gamma_2 S) V_{t+1} + \alpha_0(\gamma_3 r + \gamma_4 S) Z_{t+2} \\
& \quad + \alpha_1(\gamma_1 + \gamma_2) V_{t+1} + \alpha_1(\gamma_3 + \gamma_4) Z_{t+1} \\
& = (b_0 - g) V_t - Z_{t+1} - V_{t+1} - (b_0 - 1)(e_1 + m^*) + (e_0 - gm^*)
\end{aligned}$$

Therefore,

$$\begin{aligned} & \gamma_0(\alpha_0 + \alpha_1 r + \alpha_2 S) + \alpha_1(\gamma_1 + \gamma_2)V_{t+1} + \alpha_0[(\gamma_1 r + \gamma_2 S)V_{t+1} + (\gamma_1 + \gamma_2)V_t] \\ & + \alpha_1(\gamma_3 + \gamma_4)Z_{t+2} + \alpha_0[(\gamma_3 r + \gamma_4 S)Z_{t+2} + (\gamma_3 + \gamma_4)Z_{t+1}] \\ & = (b_0 - g)V_t - Z_{t+1} - V_{t+1} + (b_0 - 1)(e_1 + m^*) + (e_0 - gm^*) \quad (61) \end{aligned}$$

We know that  $r = -\frac{\alpha_1}{2\alpha_0} + \frac{D}{2\alpha_0}$

where  $D$  is the discriminant of the characteristic equation;  
hence

$$\begin{aligned} S &= -\frac{\alpha_1}{2\alpha_0} - \frac{D}{2\alpha_0} \\ &= r - \frac{D}{\alpha_0} \end{aligned}$$

and  $\frac{D}{\alpha_0} = 2r + \frac{\alpha_1}{\alpha_0}$

Equating coefficients in Equation 61 we obtain

$$\begin{aligned} \text{(a)} \quad \alpha_0(\gamma_1 + \gamma_2) &= b_0 - g \\ \gamma_1 + \gamma_2 &= \frac{b_0 - g}{\alpha_0} \end{aligned}$$

$$(b) \quad \alpha_1(\gamma_1 + \gamma_2) + \alpha_0[\gamma_1 r + \gamma_2 S] = -1$$

$$1 \left( \frac{b_0 - g}{\alpha_0} + \alpha_0 \left[ \gamma_1 r + \left( \frac{b_0 - g}{\alpha_0} - \gamma_1 \right) \left( r - \frac{D}{\alpha_0} \right) \right] \right) = -1$$

$$\frac{\alpha_1}{\alpha_0} (b_0 - g) + \alpha_0 \left[ \gamma_1 \left( 2r + \frac{\alpha_1}{\alpha_0} \right) - \left( r + \frac{\alpha_1}{\alpha_0} \right) \left( \frac{b_0 - g}{\alpha_0} \right) \right] = -1$$

$$\alpha_0 \left[ \gamma_1 \left( 2r + \frac{\alpha_1}{\alpha_0} \right) - r \left( \frac{b_0 - g}{\alpha_0} \right) \right] = -1$$

$$\gamma_1 D - r(b_0 - g) = -1$$

$$\gamma_1 = \frac{r(b_0 - g) - 1}{D}$$

and

$$\gamma_2 = \frac{b_0 - g}{\alpha_0} - \gamma_1$$

$$(c) \quad \alpha_0(\gamma_3 + \gamma_4) = -1$$

$$(\gamma_3 + \gamma_4) = \frac{-1}{\alpha_0}$$

$$(d) \quad \alpha_1(\gamma_3 + \gamma_4) + \alpha_0(\gamma_3 r + \gamma_4 S) = 0$$

We can show that

$$\gamma_3 D + r = 0$$

$$\gamma_3 = \frac{-r}{D}$$

and

$$\gamma_4 = -\frac{1}{\alpha_0} + \frac{r}{D}$$

We can use the same procedure to derive the coefficients in Equation 26.

#### Derivation of the Homogenous Solution

At  $t = 0$  Equation 22 becomes

$$a_0 = \phi_0 + \phi_1 \sum_0^{\infty} V_i r^i + \phi_2 \sum_0^{\infty} V_i S^i + \phi_3 \sum_0^{\infty} Z_{1+i} r^i + \phi_4 \sum_0^{\infty} Z_{1+i} S^i + \phi_5$$

Hence,

$$\phi_5 = a_0 - \phi_0 - [\phi_1 \sum_0^{\infty} V_i r^i + \phi_2 \sum_0^{\infty} V_i S^i + \phi_3 \sum_0^{\infty} Z_{1+i} r^i + \phi_4 \sum_0^{\infty} Z_{1+i} S^i]$$

(a) For a permanent unanticipated pure monetary expansion,

$$\phi_5 = 0.$$

(b) For a permanent unanticipated open market operation,

$$\phi_5 = a_0 - \phi_0$$

$$= -dZ_0 \quad \text{since } \phi_0 \text{ remains the same, but } a_0 \text{ goes} \\ \text{down by the amount purchased.}$$

(c) For a permanent anticipated pure monetary expansion,

$$\phi_5 = - \left[ \sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 s^i) (-dm) \right]$$

(d) For a permanent anticipated open market operation;

$$\phi_5 = - \left[ \sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 s^i) (-dm) + (\phi_3 r^{\tau-1} + \phi_4 s^{\tau-1}) dZ \right]$$

Having obtained solutions for  $\phi_5$  we can obtain solutions for  $\gamma_5$  through the following relationship:

$$\gamma_5 = \frac{\phi_5}{\lambda \theta_1 - (1+\lambda)}$$



APPENDIX 4. DERIVATION OF RELATIONSHIP  
BETWEEN  $\gamma_5$  AND  $\phi_5$

Equations 23 and 24 are rewritten here for convenience.

$$\lambda p_{t+1} - (1+\lambda)p_t - a_t = -V_t - (e_1 + m^*)$$

$$a_{t+1} - b_0 a_t - g p_t = g V_t - Z_{t+1} + (e_0 - g m^*)$$

This can be written in a matrix form as

$$\begin{bmatrix} p_{t+1} \\ a_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1+\lambda}{\lambda} & \frac{1}{\lambda} \\ g & b_0 \end{bmatrix} \begin{bmatrix} p_t \\ a_t \end{bmatrix} + \begin{bmatrix} -\frac{1}{\lambda} V_t & -\frac{e_1 + m^*}{\lambda} \\ -g V_t & -Z_{t+1} + e_0 - g m^* \end{bmatrix}$$

Let

$$\begin{bmatrix} p_t \\ a_t \end{bmatrix} = X^t \begin{bmatrix} \gamma_5 \\ \phi_5 \end{bmatrix}$$

hence

$$\begin{bmatrix} p_{t+1} \\ a_{t+1} \end{bmatrix} = X^t \begin{bmatrix} X \gamma_5 \\ X \phi_5 \end{bmatrix}$$

Therefore

$$X^t \begin{bmatrix} X\gamma_5 \\ X\phi_5 \end{bmatrix} + X^t \begin{bmatrix} -\frac{(1+\lambda)}{\lambda} & -\frac{1}{\lambda} \\ -g & -b_0 \end{bmatrix} \begin{bmatrix} \gamma_5 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\lambda}V_t & -\frac{e_{1+m^*}}{\lambda} \\ gV_t & -Z_{t+1} + e_0 - gm^* \end{bmatrix}$$

The complementary function is therefore given by

$$\begin{bmatrix} X & -\frac{1+\lambda}{\lambda} & -\frac{1}{\lambda} \\ & -g & X - b_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and hence,

$$(X - \frac{1+\lambda}{\lambda})(X - b_0) - \frac{g}{\lambda} = 0$$

$$\lambda X^2 - (\lambda b_0 + \lambda + 1)X + (1 + \lambda)b_0 - g = 0$$

$$\alpha_2 X^2 - \alpha_1 X + \alpha_0 = 0$$

Where  $\alpha_2$ ,  $\alpha_1$  and  $\alpha_0$  are as defined in the text. The roots of this equation are  $X_1$   $X_2$  given by

$$X_1 = \theta_1 = \frac{1}{r} \quad \text{and} \quad X_2 = \theta_2 = \frac{1}{s}$$

The relationship between  $\gamma_5$  and  $\phi_5$  is given by

$$\begin{bmatrix} (\theta_1 - \frac{1+\lambda}{\lambda}) & -\frac{1}{\lambda} \\ -g & \theta_1 - b_0 \end{bmatrix} \begin{bmatrix} \gamma_5 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and hence

$$\gamma_5(\theta_1 - \frac{1+\lambda}{\lambda}) - \frac{1}{\lambda}\phi_5 = 0 \quad (62)$$

$$-g\gamma_5 + \phi_5(\theta_1 - b_0) = 0 \quad (63)$$

Let us note that  $\theta_1 - \frac{1+\lambda}{\lambda} < 0$ , hence if  $\gamma_5 > 0$  it implies that  $\phi_5 < 0$  and if  $\gamma_5 < 0$  it implies that  $\phi_5 > 0$ . That is  $\gamma_5 \cdot \phi_5 < 0$ . From Equation 63 we have

$$\phi_5 \left[ \frac{\theta_1 - b_0}{g} \right] = \gamma_5$$

Therefore,

$$\phi_5^2 \left[ \frac{\theta_1 - b_0}{g} \right] = \gamma_5 \phi_5$$

Hence, we conclude that  $\frac{\theta_1 - b_0}{g} > 0$  and therefore from Equation 63, if  $\gamma_5 > 0$  it implies that  $\phi_5 < 0$  and if  $\gamma_5 < 0$  it implies that  $\phi_5 > 0$ . From Equation 62, we can therefore make the following conclusions: if  $\phi_5 > 0$ , that is if

$$\sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 S^i) dm - (\phi_3 r^{\tau-1} + \phi_4 S^{\tau-1}) dZ > 0$$

then

$$\gamma_5 = \frac{\theta_5}{\lambda \theta_1 - (1 + \lambda)} < 0$$

And secondly, if  $\phi_5 < 0$  that is if

$$\sum_{i=0}^{\tau-1} (\phi_1 r^i + \phi_2 S^i) dm - (\phi_3 r^{\tau-1} + \phi_4 S^{\tau-1}) dZ < 0$$

then

$$\gamma_5 = \frac{\phi_5}{\lambda \theta - (1 + \lambda)} > 0$$